Unsteady one-dimensional state solidification and melting of PCM with heat generation

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ABSTRACT

Numerical analysis of one-dimensional transient solidification and melting of a slab with an uniform volumetric energy generation presented. A fully implicit finite difference method is used to resolve the dimensionless energy equations in each case. The effect of the dimensionless heat generation coefficient on the dimensionless temperature in both fields and interface location is presented. For the melting case, the evolution of the total melting time is also exposed.

KEYWORDS: PCM, Solidification, Melting, Volumetric energy generation

NOMENCLATURE

C	Specific heat, J/kg °C	t	Time, s
h	Latent heat of fusion, J.kg^{-1}	T	Temperature, °C
k	Thermal conductivity, W/m°C	T_f	Fusion temperature, °C
L	Thickness of the slab, m	T_0	Surface temperature, °C
Q	Volumetric heat generation, W/m^3	x	Spatial variable, m
St	Stefan number		

Greek Symbols

α	Thermal Diffusivity, m^2.s^{-1}
β	Dimensionless heat generation coefficient
θ	Dimensionless temperature
τ	Dimensionless time
τ_0	Total melting time

Subscripts

f	Fusion
i	interface
L	Liquid
s	Solid

1. INTRODUCTION

Transient heat-transfer problems involving melting or solidification are generally referred to as phase-change or moving boundary problems. Sometimes, they are referred to as Stefan problems, with reference to the pioneering work of Stefan around 1890. Phase change problems have numerous applications in such areas as the making of ice, the freezing of food, the solidification of metals in castings and recently new protective clothing are being developed that provide significant enhancements in thermal storage and comfort using encapsulated phase change materials. One of prospective of storing solar energy for example is the application of phase change materials (PCMs). In spite of the fact that a great attention has been done on phase change problems [1-4], few articles deal with the effect of volumetric heat generation on them. In the literature, there are few analytic solutions of this kind of problem because of the non-linearity of the phase change materials governing equations and other complicating factors. In compensation, numerous approximate analytical solutions exist [5-7]. Indeed, Jiji et al. [5] used a quasi-static one approach to study one dimensional solidification and melting of a slab with uniform energy generation. In the reference [6], a same approach valid for Stefan numbers less than, is employed to examine equations governing the motion of a phase change front for materials which generate internal heat in the case of cylindrical, spherical, plane wall and semi-infinite geometries. Kalaiselvam et al. [7] analyzed an experimental and analytical investigation of solidification and melting characteristics of PCMs inside cylindrical encapsulation. Chan et al. [1] described a phase change model where internal melting was introduced by radiative transfer in semi-transparent materials. Chan and Hsu [8] used finite difference techniques and the enthalpy method to study the phase change of materials with internal heat generation, specifically the mushy zone of along the interface.

The present work concentrates on the effect of a uniform volumetric energy generation on one-dimensional transient solidification and melting Phase Change Material (PCM).

2. HYOTHESES

Numerous hypotheses were made to simplify the present analysis (solidification and melting). It was assumed that the phase change occurred at a single fusion temperature, which enabled us to model the phase change front as sharp boundary between the solid and liquid phases. It was also assumed that there was no convection heat transfer in the liquid, so that heat is transferred solely by conduction. Finally, it was assumed that the volumetric heat generation was constant, uniform, and equal in the both phases.

3. SOLIDIFICATION ANALYSIS

The phase solidification of a slab of finite thickness L is considered (figure 1). It is initially at the temperature T_i which is above the fusion temperature . The surface $x=0$ is suddenly maintained at $T_0 < T_f$ whereas the surface $x=L$ is insulated. Simultaneously, energy is generated throughout the system at constant volumetric rate q. A solid-liquid interface forms instantaneously at $x=0$ and propagates through the liquid phase. The medium is assumed to be homogeneous and isotropic with constant and identical thermophysical properties in both phases and the motion of the liquid phase is neglected.

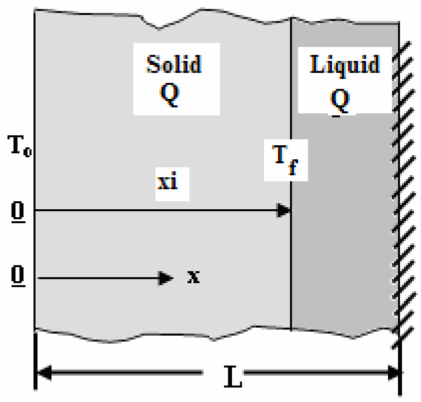

Figure 1. Physical model and coordinates system.

The following dimensionless variables [5; 9] are used:

- in the solid phase, $0 \le x \le xi$: $\eta = \dfrac{x}{xi}$ (1a)
- in the liquid phase, $xi \le x \le L$: $\eta = 1 + \dfrac{(x - xi)}{(L - xi)}$ (1b)
- $\tau = St\dfrac{\alpha}{L^2}t$, $\theta_\xi = \dfrac{T_\xi - T_0}{T_f - T_0}$ ($\xi = L$ or s or initial), $St = \dfrac{C(T_f - T_0)}{h}$, $\beta = \dfrac{QL^2}{k(T_f - T_0)}$ (2a-d)

The dimensionless energy equations in both phases are given by:

$$\frac{\partial^2 \theta_s}{\partial \eta^2} + \frac{xi^2}{L^2}\beta = St\frac{xi^2}{L^2}\frac{\partial \theta_s}{\partial \tau}, \tag{3a}$$

$$\frac{\partial^2 \theta_L}{\partial \eta^2} + \frac{(L - xi)^2}{L^2}\beta = St\frac{(L - xi)^2}{L^2}\frac{\partial \theta_L}{\partial \tau} \tag{3b}$$

The following dimensionless interface energy balance will be used to develop the differential equation governing the motion of the phase change front.

$$\left.\frac{\partial\theta_s}{\partial\eta}\right|_{\eta=1} - \frac{xi}{(L-xi)}\left.\frac{\partial\theta_L}{\partial\eta}\right|_{\eta=1} = \frac{1}{2L^2}\frac{dxi^2}{d\tau} \qquad (4)$$

The dimensionless initial and boundary conditions are respectively:

- $\theta_L(\eta,0)=\theta_{initial}$, $xi(0)=0$ (5a-b)
- At the wall, $\eta=0$: $\theta_s(0,\tau)=0$ (6a)
- At the solid-liquid interface, $\eta=1$: $\theta_S(1,\tau)=\theta_L(1,\tau)=1$ (6b)
- At the insulated wall, $\eta=2$: $\frac{\partial\theta_L}{\partial\eta}=0$ (6c)

4. MELTING ANALYSIS

The identical problem with the material initially solid is considered. The surface at $x=0$ is suddenly exposed to a temperature T_0 which is higher than the phase change temperature ($T_0 > T_f$). Melting of PCM starts from the previous surface and the interface moves towards the right side of the system. Because of the temperature rise and the presence of heat generation, solid phase will result in partial melting and hence solid-liquid region is formed instead of the solid phase. It assumed that the mixture forms immediately and the temperature of the solid phase increases instantaneously to the melting temperature. Mixture will be in its fusion temperature but the proportion of the liquid in the mixture rises due to heat generation and sensible heat addition from the warm surface. In the dimensionless formulation, the heat equation for the liquid phase is given by:

$$\frac{\partial^2\theta_L}{\partial\eta^2} - \frac{xi^2}{L^2}\beta = St\,\frac{xi^2}{L^2}\frac{\partial\theta_L}{\partial\tau} \qquad (7)$$

where: $\eta=\frac{x}{xi}$ for $0\le x\le xi$, $\tau = St\frac{\alpha}{L^2}t$, $\theta_L=\frac{T_L-T_0}{T_f-T_0}$, $St=\frac{C(T_0-T_f)}{h}$, $\beta=\frac{QL^2}{k(T_0-T_f)}$

The dimensionless initial and boundary conditions include respectively:

- $\theta_L(\eta,0)=\theta_{initial}$ $xi(0)=0$ (8a-b)
- At the wall, $\eta=0$: $\theta_L(0,\tau)=0$ (9a)
- At the solid-liquid interface, $\eta=1$: $\theta_L(1,\tau)=1$ (9b)

Assuming no heat is transferred through the mixture by conduction and in taking into account the conservation energy of the mixture, the interface energy equation is modified with an addition of a factor $\gamma=\beta\tau$ which defines the mass proportion of liquid in the mixture [5, 7]:

$$\left.\frac{\partial\theta_L}{\partial\eta}\right|_{\eta=1} = \frac{(1-\beta\tau)}{2L^2}\frac{dxi^2}{d\tau} \qquad (10)$$

5. SOLUTIONS METHODS

A fully implicit finite difference method is used to resolve the dimensionless energy equations of solidification and melting (3a-b and 7). Spatial and temporal grids are chosen to be uniform. Dimensionless discretised energy equations form a tridiagonal matrix which can be efficiently solved by the TDMA algorithm. Next, the interface location xi is numerically determined by using the dimensionless interface energy balances (4 and 10) and the secant method.

6. RESULTS AND DISCUSSION

6.1 Solidification

The thermophysical properties used here are those of a slab of nuclear material UAI_x [5] and the Stefan number is $St=0.09133$. The parameter values, to which this computation is performed, are mentioned in the figure captions in particular β=0.0; 0.8; 1.6; 2; 5.0 and 10.0.

The results present the dependence of the interface position and temperature profiles on the scaled heat generation coefficient β. Then; figure 2 depicts transient distributions of the dimensionless temperature in both fields. It is seen from this figure that the discontinuity in the temperature gradient at the interface disappears when the parameter β becomes higher. Furthermore, the temperature rises monotonously and depends on the parameter β. This augmentation is less important for $0 \le \beta \le 2$ than the case where $2 \le \beta \le 10$. On the other hand, the effect of the dimensionless heat generation on the temporal evolution of the interface position is presented in the figure 3. We note that increasing heat generation provokes the slow motion of the interface position and for $\beta > 2$ the steady state solidification takes place. From the steady state interface position $\left(\frac{xi}{L}\right)_\infty$ (figure 4) we can see that the solidification is total in the first region 1 ($\left(\frac{xi}{L}\right)_\infty \approx 1$) whereas it's partial in the region 2 ($\left(\frac{xi}{L}\right)_\infty$ <1). These results are in good agreement with those of the reference [5]. According to this later reference, the special case of $\beta = 2$ corresponds to a unique value of energy generation for which the study state interface is at the insulated surface. In this case the insulated surface is at the fusion temperature.

6.2 Melting

Now let us consider the melting case where a slab of ice with the Stefan number $St = 0.1007$ is studied [5]. This case is characterized by a pure liquid phase and a solid-liquid mixture at the fusion temperature. Unlike solidification, both surface heat transfer and heat generation affect the total melting time. So, the effect of dimensionless heat generation β on the transient interface position is illustrated in figure 5. It is seen that an increase in this parameter accelerates melting and shortens total melting time $\tau_{\text{total melting}}$. This fact is also highlighted in figure 6 which shows that the total melting time for the case of no energy generation is $\tau_{\text{total melting}} = 0.5$. This value decreases in increasing the dimensionless heat generation β.

5. CONCLUSION

One-dimensional transient solidification and melting of a PCM with volumetric heat generation has numerically been studied for the Stefan number less than one. Differential equations modelling the motion of the phase change front have been derived. Both solidification and melting solutions are governed by the dimensionless heat generation β.
To better understand the behavior of a phase change material front in a material, this study can be extended to account for solid phase properties that differ from the liquid phase and the model can also be applied to variable heat generation.

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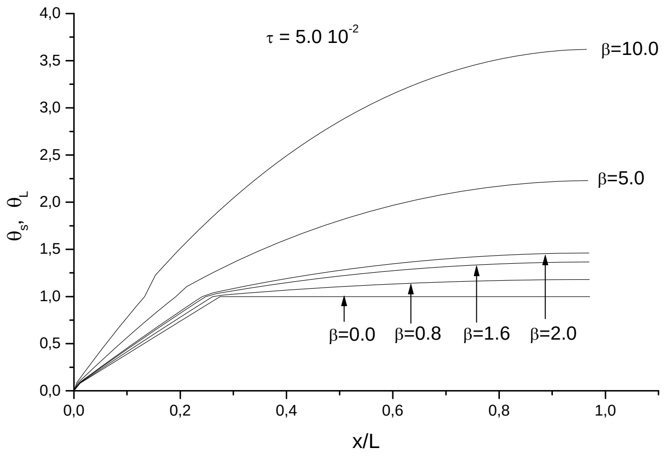

Figure 2: Dimensionless temperature distribution in both fields.

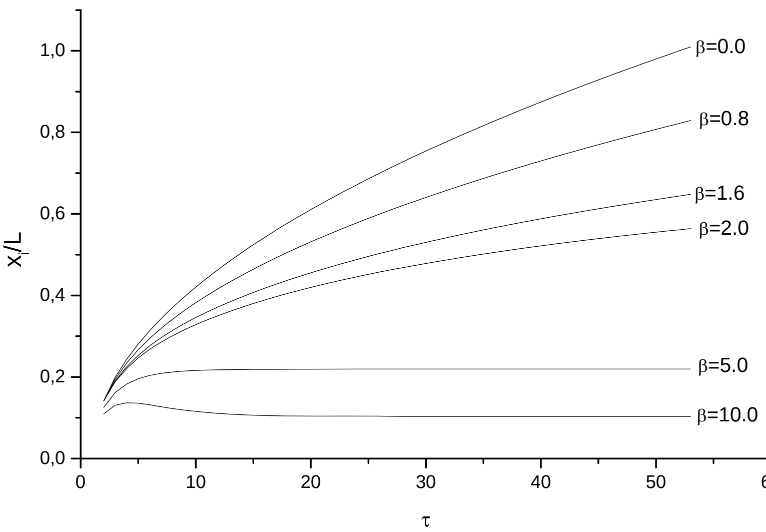

Figure 3: Dependence of the interface position on the scaled heat generation coefficient.

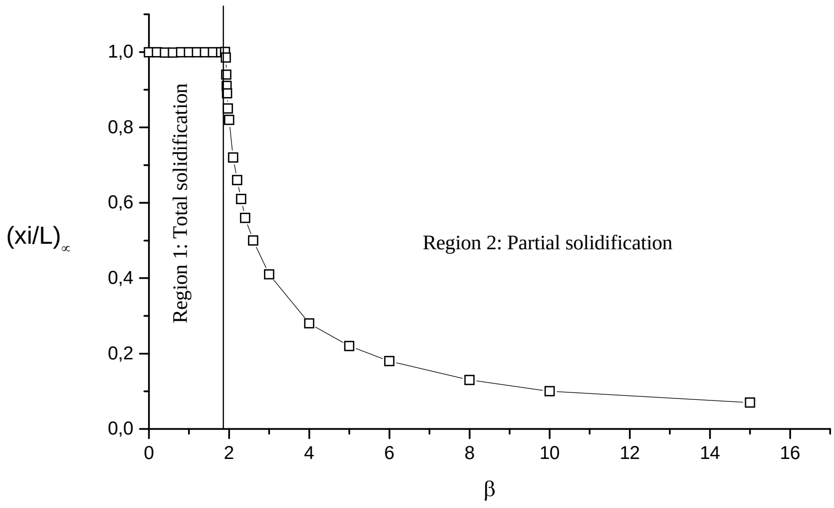

Figure 4: Effect of the heat generation on the phase change problem.

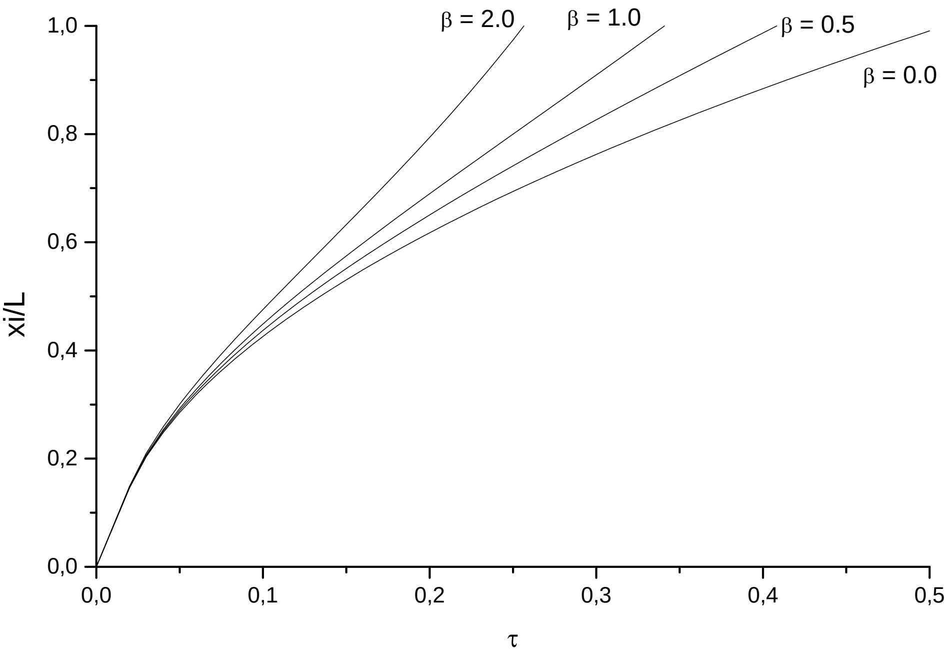

Figure 5: Influence of the dimensionless heat generation on the transient interface position.

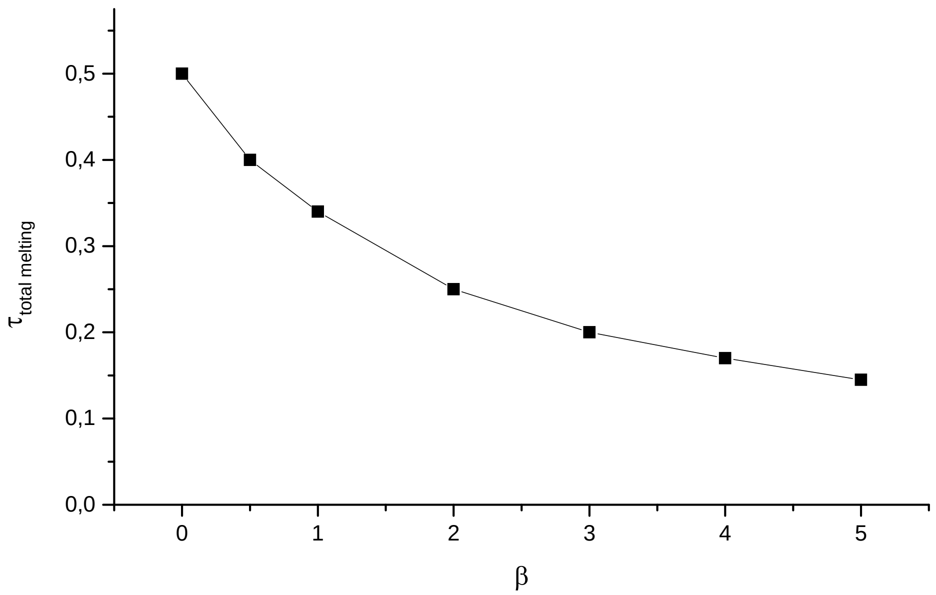

Figure 6 : Energie generation effect on total melting.