STUDY OF THE EFFECT OF AN EXTERNAL MAGNETIC FIELD ON THE ENTROPY GENERATION IN THREE-DIMENSIONAL NATURAL CONVECTION.

L. Kolsi*, A. Abidi, M.N. Borjini et H. Ben Aïssia
Unité de métrologie en mécanique de fluide, Ecole nationale d'ingénieur de Monastir
lioua_enim@yahoo.fr

RESUME
Cette étude présente l'effet d'un champ magnétique externe sur l'entropie générée dans le cas de la convection naturelle tridimensionnelle d'un métal liquide. Le nombre de Prandtl est fixe à Pr=0,026, le nombre de Rayleigh est fixe à $Ra=10^4$ et le nombre de Hartmann varie entre 0 et 150. Le coefficient d'irréversibilité varie entre 10^{-4} et 10^{-1}. Un intérêt particulier est donné à l'étude de la distribution 3D de l'entropie générée et à l'effet du champ magnétique sur les différents types d'irréversibilités.

1. INTRODUCTION:

Many thermodynamic systems like the heat exchangers, turboshaft engines, electrothermics, porous media are the subject of the irreversibility phenomena due to heat gradients, friction effects, diffusion and Joule effect etc… The analysis of the second law of thermodynamic has recently gained an important attention in order to minimize these irreversibilties. However there are few works concerning entropy generation in confined natural convection situations.

Among a small number of works relating to entropy generation in the case of magnetohydrodynamic natural convection in confined enclosure, it figures the study effectuated by Mahmud and Fraser [1] in the case of saturated porous cavity. In the absence of the magnetic force, the rate of entropy generation is relatively higher close to the two vertical walls while this rate falls according to Hartmann number.

Entropy generation in magnetohydrodynamic problems is more developed for flow in channels. Ibáñez et al. [10] applied the method of minimization of entropy generation to optimize a magnetohydrodynamic flow between two infinite parallel walls having a finished electric conductivity. The authors showed that the generation of entropy reaches a minimum when the walls are cooled in an asymmetrical way. Also Mahmud & al. [11], Tasnim et al. [12] and Aïboud-Saouli [13] carried out an analysis to study the first and the second laws of thermodynamics of a flow of mixed laminar convection inside a vertical channel under the action of a transverse magnetic field.

Within this framework we study the effect of a magnetic field on these losses of energy in the case of the three-dimensional natural convection.

2. MATHEMATICAL FORMULATION AND NUMERICAL MODEL

Figure 1 schematizes the configuration considered: the left and right walls are differentially heated, the other walls are considered adiabatic, and a homogeneous magnetic field expressed by $B_0.\vec{e}_B$ is imposed perpendicular to the heated walls. All the walls of the cavity are considered insulating electrically.

As numerical method we had recourse to the vorticity-vector potential formalism $(\vec{\psi}-\vec{\omega})$.

The setting in equation is described with more detail in the article of Kolsi et al. [6].

In the presence of a magnetic field the generated entropy is written in the following form:

$$S'_{gen} = -\frac{1}{T'^2}.\vec{q}'.\vec{\nabla}T' + \frac{\mu}{T'}.\phi' + \frac{1}{T'}.\left(\vec{J}'-\rho_e.\vec{V}'\right)\left(\vec{E}'+\vec{V}'\times\vec{B}'\right)$$

The first term represents the generated entropy due to temperature gradient, the second that due to the friction effects and the last that due to the presence of the magnetic field. $\rho_e.\vec{V}'$ being negligible and $\vec{q}' = -k.\vec{grad}T'$. The dissipation function is written in incompressible flow as:

$$\phi' = 2\left[\left(\frac{\partial V'_x}{\partial x'}\right)^2 + \left(\frac{\partial V'_y}{\partial y'}\right)^2 + \left(\frac{\partial V'_z}{\partial z'}\right)^2\right] + \left(\frac{\partial V'_y}{\partial x'} + \frac{\partial V'_x}{\partial y'}\right)^2 + \left(\frac{\partial V'_z}{\partial y'} + \frac{\partial V'_y}{\partial z'}\right)^2 + \left(\frac{\partial V'_x}{\partial z'} + \frac{\partial V'_z}{\partial x'}\right)^2 \quad (7)$$

From where the generated entropy is written:

$$S'_{gen} = \frac{k}{T'}\left[\left(\frac{\partial T'}{\partial x'}\right)^2 + \left(\frac{\partial T'}{\partial y'}\right)^2 + \left(\frac{\partial T'}{\partial z'}\right)^2\right] + 2\left[\left(\frac{\partial V'_x}{\partial x'}\right)^2 + \left(\frac{\partial V'_y}{\partial y'}\right)^2 + \left(\frac{\partial V'_z}{\partial z'}\right)^2\right] + \left(\frac{\partial V'_y}{\partial x'} + \frac{\partial V'_x}{\partial y'}\right)^2 + \left(\frac{\partial V'_z}{\partial y'} + \frac{\partial V'_y}{\partial z'}\right)^2 + \left(\frac{\partial V'_x}{\partial z'} + \frac{\partial V'_z}{\partial x'}\right)^2 \quad (8)$$

$$+ \frac{1}{T'}\frac{1}{\sigma_e}\left(J'^2_x + J'^2_y + J'^2_z\right)$$

After adimensionalisation one obtains generated entropy number (dimensionless local generated entropy) which is written in the following way:

$$N_s = S'_{gen}\frac{1}{k}\left(\frac{lT_m}{\Delta T}\right)^2 \quad (9)$$

From where:

$$N_s = \left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2\right] + \varphi.\left\{2\left[\left(\frac{\partial V_x}{\partial x}\right)^2 + \left(\frac{\partial V_y}{\partial y}\right)^2 + \left(\frac{\partial V_z}{\partial z}\right)^2\right] + \left[\left(\frac{\partial V_y}{\partial x} + \frac{\partial V_x}{\partial y}\right)^2 + \left(\frac{\partial V_z}{\partial y} + \frac{\partial V_y}{\partial z}\right)^2 + \left(\frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x}\right)^2\right]\right\} \quad (10)$$

$$+ \varphi.Ha^2\left(J_x^2 + J_y^2 + J_z^2\right)$$

With $\varphi = \frac{\mu\alpha^2 T_m}{l^2 k \Delta T^2}$ is the irreversibility coefficient.

The first term of N_s represents the local irreversibility due to the temperatures gradients, it is noted $N_{S\text{-}th}$. The second term represents the contribution of the viscous effects in the irreversibility it is noted $N_{S\text{-}fric}$ and the third term represents the generated local entropy due to the Joule effect it is noted $N_{S\text{-}J}$. N_s give a good idea on the profile and the distribution of the generated local dimensionless entropy. The total dimensionless generated entropy is written:

$$S_{tot} = \int_v N_s dv = \int_v \left(N_{s-th} + N_{s-fr} + N_{s-J}\right)dv = S_{th} + S_{fr} + S_J \quad (11)$$

3. RESULTS AND DISCUSSION:

In this study, the Prandtl number is fixed at Pr=0,026 relating to a molten metal, the Rayleigh number is fixed at $Ra=10^4$ and the Hartmann number lies between 0 and 150. The irreversibility coefficient lies between 10^{-4} and 10^{-1}. A particular interest is given to the study of the 3D distribution of the generated entropy and to the effect of the magnetic field on different types of the irreversibilities.

3.2. VARIOUS TYPES OF IRREVERSIBILITIES

Figure 1 represents the variation of the total generated entropy according to Hartmann number for different irreversibility coefficient. It is noticed that for $\varphi = 10^{-1}$, $\varphi = 10^{-2}$ and $\varphi = 10^{-3}$, the generated entropy grows slightly then decrease by increasing Ha. For $\varphi = 10^{-4}$ the growing zone does not exist any more. This attenuation is explained by the magnetic damping of the flow.

Figures 2, presents for various irreversibility coefficients the variation of S_{th}, S_{fric}, S_J and S_{tot} according to Ha. One notices that S_{th} and S_{fric} decrease according to Ha, but S_J presents a maximum. From where the maximum in the variation of the total generated entropy according to Ha is due to the dissipation by Joule effect. This result is also met in the 2D chanal flow [10].

By analyzing these figures, one notices that for $\varphi = 10^{-1}$ and $\varphi = 10^{-2}$ the generated entropy due to the thermal transfer is negligible compared to that due to the viscous effects and the Joule effect. For $\varphi = 10^{-3}$, one notices that S_{th} becomes of the same order of magnitude as S_{fric} and S_J. For $\varphi = 10^{-4}$ the generated entropy due to the variation in temperature becomes dominant.

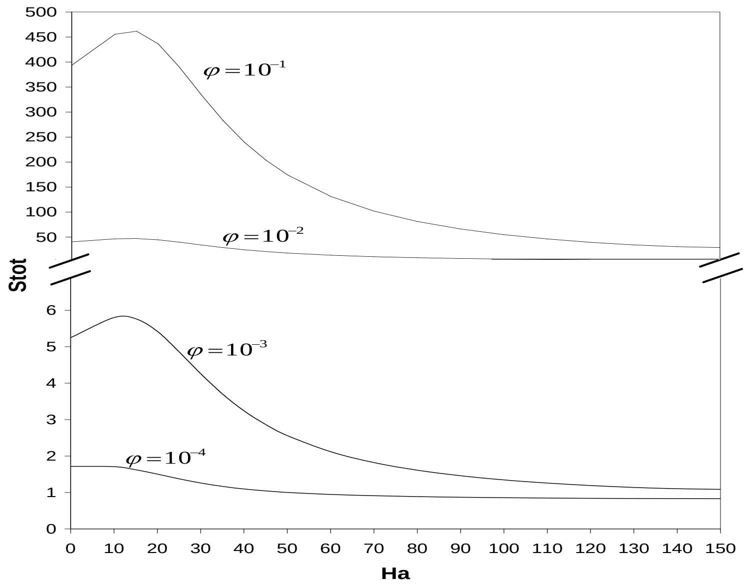


Figure 1. Variation of the total generated entropy according to the Hartmann number

Figures 2 also show that the effect of the magnetic field is more considerable on S_{fric} and S_J that on S_{th} and show that S_J presents a maximum for all the values of φ . It is also noticed that the maximum of S_J and S_{fric} occurs for $15 \leq Ha \leq 20$ for all considered values of φ .

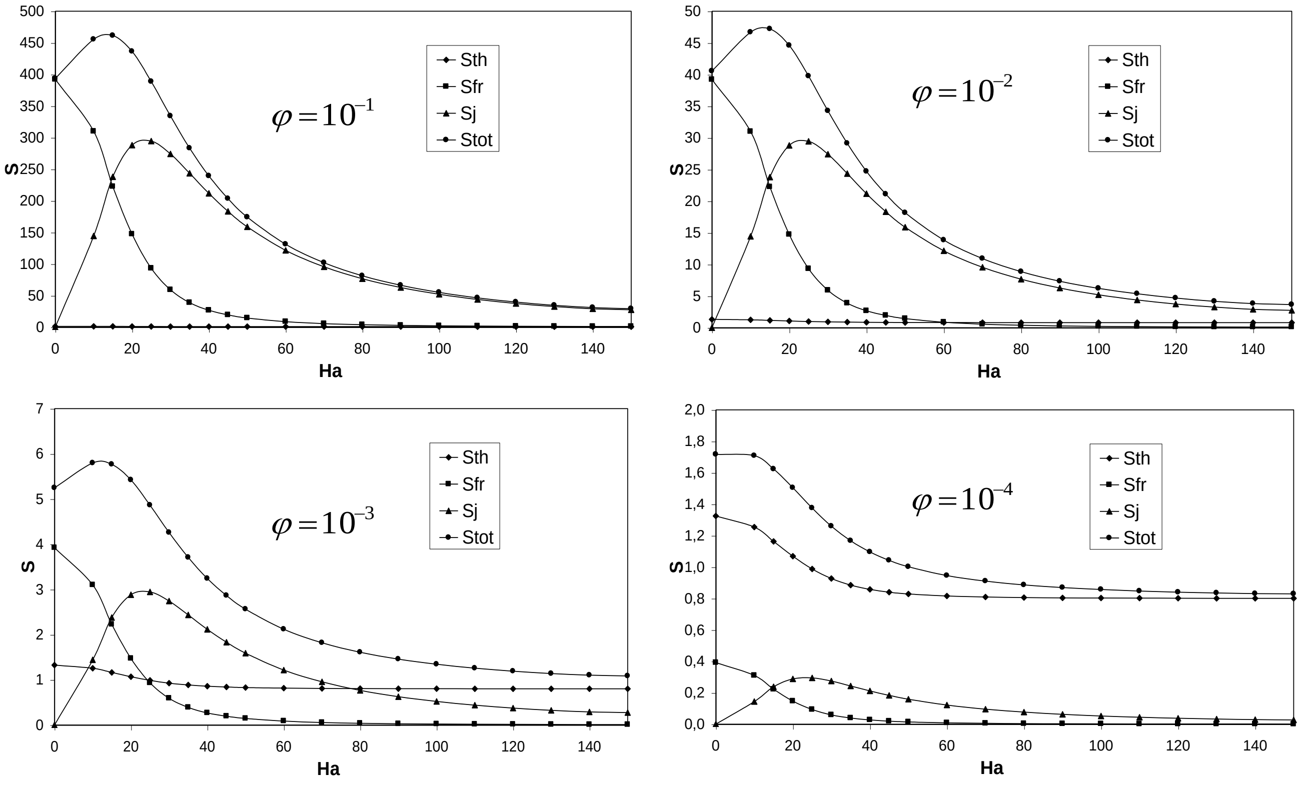


Figure 2. Variation of generated entropy according to Ha.

3.3. 3D DISTRIBUTION OF THE IRREVERSIBILITIES

In order to analyze the three-dimensional aspect of the entropy generation we traced for two values of φ the total generated entropy for Ha=0, Ha=50 and Ha=100 (figure 3). The 3D behavior is more important for Ha=0 for both $\varphi=10^{-1}$ and $\varphi=10^{-4}$. For $\varphi=10^{-1}$ the generated entropy occurs principally near the active walls for both moderately and highly damped flow. As predicted and for $\varphi = 10^{-4}$ and Ha = 0, the creation of entropy is mainly localized near the bottom of the hot surface and the top of the cold surface.

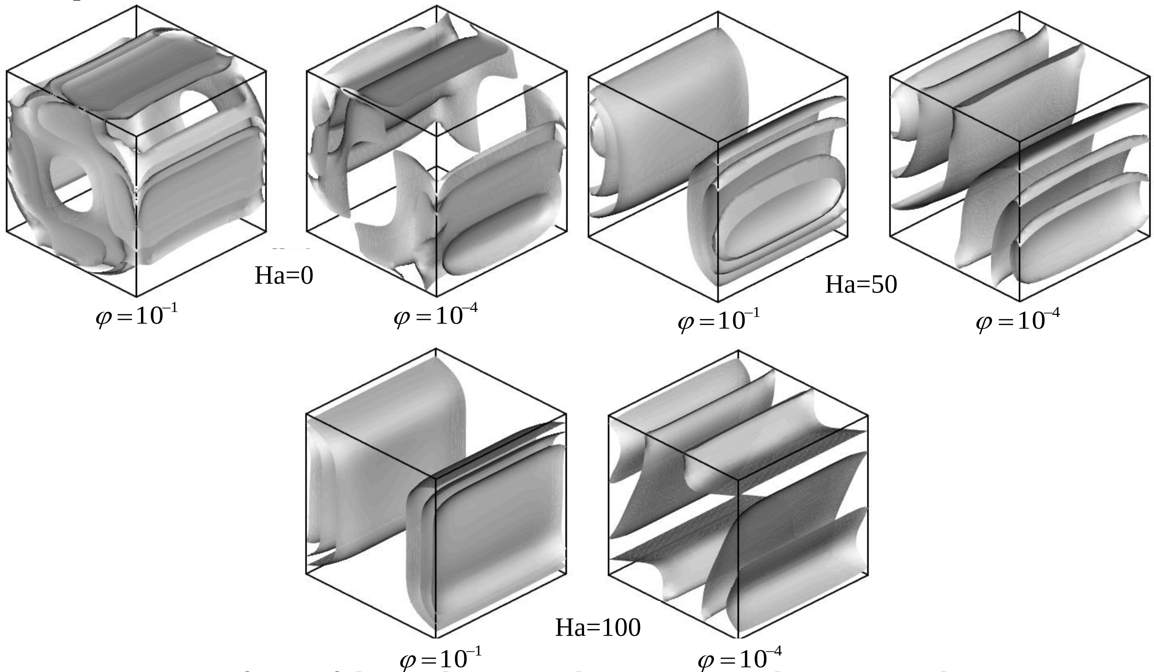


Figure 3. Iso-surfaces of the total generated entropy according to φ and Ha : Ra=10000

Figure 4 presents for Ha=0 and Ha=50 the distribution of the local total generated entropy in the z=0,5 (in continued lines) and z=0,9 (in dashed lines) plans for different values of the irreversibility coefficient . This figure confirms that the magnetic field limits the 3D character of the distribution of the generated entropy for all value of φ . This 3D behavior is clear for $\varphi = 10^{-3}$ and Ha=0.

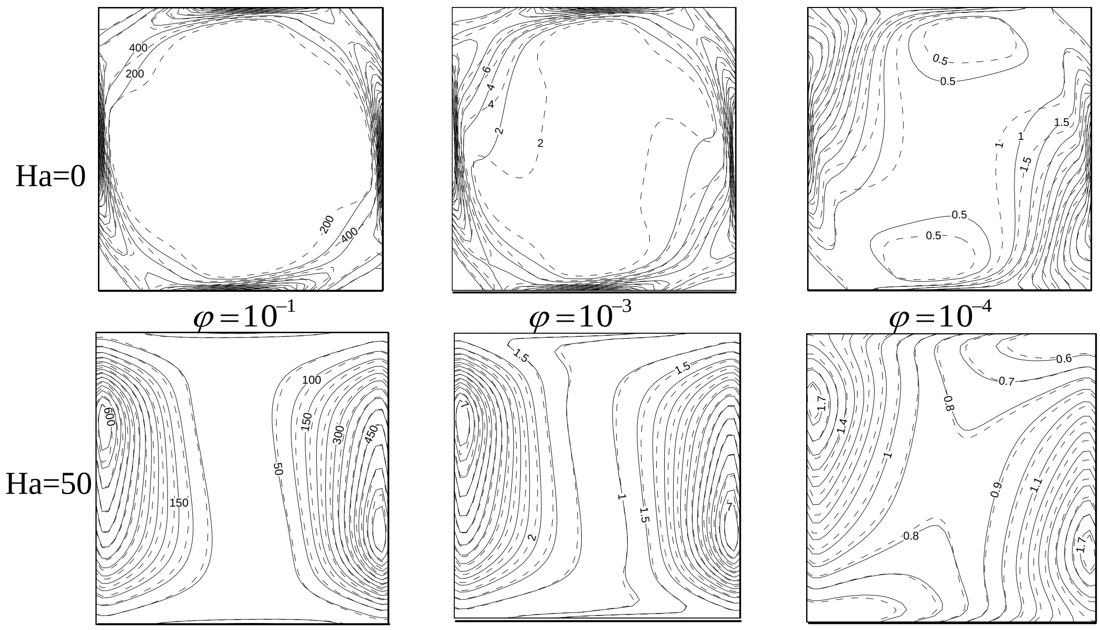
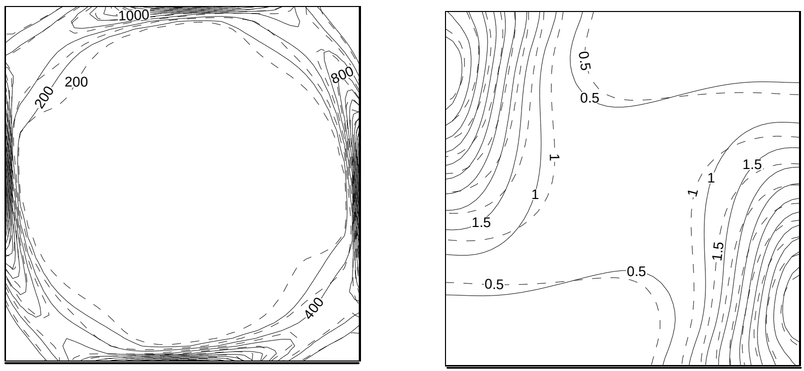


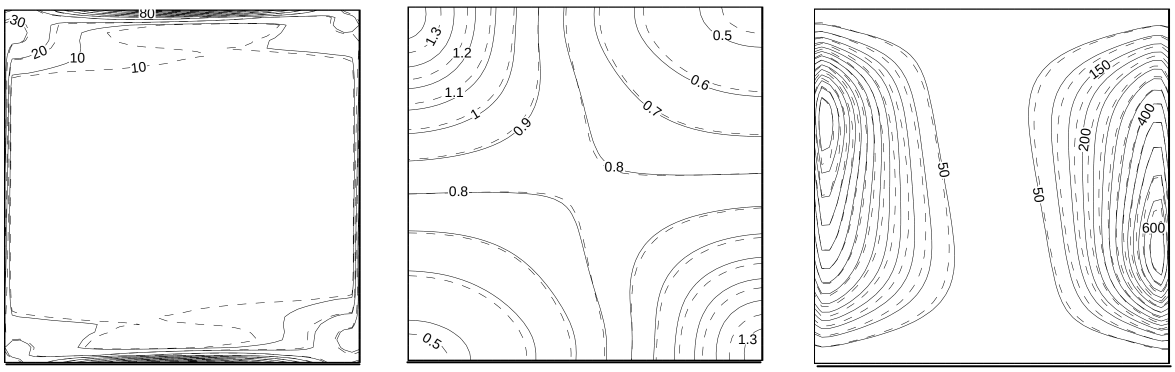
Figure 4. Contours of total generated entropy; continuous: z=0.5 plan; dashed: z=0.9 plan

Figures 5 and 6, represents respectively for Ha=0 and Ha=50 a decomposition of the distribution of total generated entropy for $\varphi = 10^{-1}$. For Ha=0, S_{tot} is broken up into S_{th} and S_{fric}. For Ha=50, S_{tot} is broken up into S_{th}, S_{fric} and S_J. These figures show that for Ha=0, the distribution of S_{tot} is very similar to the iso-entropies due to frictions what indicates the predominance of the irreversibility due to the viscous effects. By decreasing φ the distribution approaches with the iso-entropies due to temperature gradient what indicates the predominance of the irreversibility due to the thermal transfer. The 3D character exists for both S_{th} and S_{fric}. This is also noticed that for low coefficient of irreversibility the generated entropy covers all the z=0,5 plan. By increasing the coefficient of irreversibility, the generated entropy concentrates (locates itself) along the walls of the cavity. The important generation of S_{fric} is located near the center of the faces. The generation of S_{th} is near the top corner of the hot wall and the bottom of the cooled wall. For Ha=50 the generated entropy is distributed on all the cavity and nonlocalised meadows of the walls even for $\varphi = 10^{-1}$, which implies that the magnetic field is opposed to the boundary layer phenomenon met for the great Rayleigh numbers. It is also noticed that iso-contours of entropy in the z=0,5 and z=0,9 plans are almost confused. This can be explained by the bi-dimensionalisation of the flow under the effect of the magnetic field. The 3D behaviour of the distribution of the generated entropy is important for only the S_{fric}. The maximum of S_J is located in the region near the center of active walls. Figures 10 and 11 also show that the 3D character is more pronounced for the entropy generated due to the viscous effects than the other types of irreversibilities.



(a) (b)

Figure 5. Contours of generated entropy for Ha=0 and $\varphi = 10^{-1}$: (a) due to friction; (b) due to temperatures variations; in continuous lines: z=0,5 plan; in dashed lines: z=0,9 plan



(a) (b) (c)

Figure 6. Contours of generated entropy for Ha=50 and $\varphi = 10^{-1}$: (a) due to friction; (b) due to temperatures variations; (c) due to the Joule effects; in continuous lines: z=0,5 plan; in broken lines: z=0,9 plan.

4. CONCLUSION

A study concerning the effect of the presence of a magnetic field on the production of entropy in the case of the natural convection of a molten metal in a cubic cavity was conducted. This field is applied orthogonally to the isothermal vertical and opposing walls. Some conclusions can be resumed:

- In the presence of a magnetic field the generated entropy is distributed on all the cavity and non localized in the vicinity of the walls, which implies that the magnetic field is opposed to the boundary layer phenomenon met for the great Rayleigh numbers.
- The entropy generated by Joule effect presents a maximum for $10<Ha<20$ for all considered values of irreversibility coefficient. This range must be avoided when aiming the magnetic damping of the flow.
- The magnetic field limits the 3D character of the distribution of the generated entropy. This character is more pronounced for the entropy generated due to viscous effects.
- The entropy generated by friction and Joule effect is more influenced by the magnetic field than that generated by thermal dissipation.

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