NUMERICAL SIMULATION OF DOUBLE-DIFFUSIVE MIXED CONVECTION WITHIN A HORIZONTAL ANNULUS EMBEDED WITH AN ANISOTROPIC POROUS MATRIX

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ABSTRACT
A numerical investigation of double diffusive laminar mixed convection within a two dimensional, horizontal annulus filled with an anisotropic porous medium has been carried out. Anisotropy in permeability is considered with principal axes oriented at a constant angle with respect to vertical direction. The inner and outer boundaries of the annulus are held at different uniform temperature with the inner one being warmer and solutal. The outer cylinder is rotating in the counter clockwise direction to introduce the forced convection effect. The general model DBF (Darcy, Brinckman, and Forchheimer) is used.
The set of equations, governing the physical situation is solved by the control volume method. The streamlines, isotherms and isoconcentrations as well as both local and average Nusselt and Sherwood numbers were studied using several dimensionless groups including the permeability ratio K^*, the Rayleigh number (Ra), the Reynolds number (Re), as well as the buoyancy ratio N.

RÉSUMÉ
Ce travail consiste en l'analyse de l'effet de l'insertion d'une matrice poreuse anisotrope sur le comportement hydrodynamique, thermique et massique d'un écoulement dans un espace annulaire horizontal dont le cylindre extérieur est tournant. La méthode des volumes finis a été utilisée pour résoudre les équations de conservation du problème. Les nombres adimensionnels de Reynolds (Re), Darcy (Da), Rayleigh (Ra) ainsi que le rapport des perméabilités K* et le rapport des forces de flottaison N ont été retenus pour présenter les résultats qui portent sur le nombre de Nusselt (et/ou de sherwood), les lignes de courant ainsi que les isothermes (et/ou les isoconcentrations).

Nomenclature
C: Dimensionless concentration
$C^`$: dimensional concentration
C_f: inertial coefficient
Da : Darcy number
$\overline{\overline{K}}$: *permeability tensor*
K*: permeability ratio (K1/K2)
Le : Lewis number
Nu : Nusselt number
N: Buoyancy ratio
Ra : Rayleigh number
Re: Reynolds number
Sh: Sherwood number
T: dimensional temperature
$\vec{V}(V,U)$:velocity field

Greek letters
Ω : Angular velocity (rad/s)
γ: Position angle of the permeability principal axes.
θ: dimensionless temperature

Subscripts:
o : outer
i: inner
m: Mean or average

1. INTRODUCTION
The problem of double diffusive convection flow in confined spaces (with or without a porous medium insertion) has attracted many researchers due to its many engineering applications such semi conductor device fabrications, crystal growth, atmospheric and oceanic flow, etc.
Yoo [1998] performed a numerical study for a two dimensional mixed convection within a horizontal concentric annulus with a cooled rotating outer cylinder. He found that flow

patterns can be categorized into three basic types according to the number of eddies which can appear in the annulus: two eddies, one eddy and no eddy flows.
Amiri and Khanafer [2006] have studied the double diffusive mixed convection in a horizontal annulus with hot and solutal inner cylinder and rotating outer cylinder. The Galerkin finite element method was used to solve the set of the governing equations based on vorticity stream function formulation. Their results showed that high Lewis number values significantly improve the mass transfer rate, whereas it has insignificant impact on heat transfer rate. Teamah [2007] investigated numerically the same problem than Amiri and khanafer [2006] but with inner rotating cylinder.
For a horizontal annulus filled with a porous medium, Khanafer and Chamkha [2003] conducted a numerical study for steady laminar mixed convection within two concentric cylinders in the presence of internal heat generation. Their results showed that an increase of Reynolds number has a significant effect on the flow patterns within the annulus with respect to two-eddy, one eddy and no eddy flows. Natural convection in a porous medium with anisotropic permeability was considered by Aboubi et al. [1998], who conducted a numerical investigation in a horizontal annulus with the outer cylinder being warmer than the inner one. Their results indicate that the angle of inclination of the principal axes of permeability has a significant effect on the flow and temperature fields in the annulus. They observed a net circulating flow around the annulus with the maximum value reached for an inclination angle approximately of 40° with respect to the vertical diameter.
In this study, a two dimensional double diffusive mixed convection problem in a horizontal concentric annulus filled with an anisotropic porous matrix is investigated. Anisotropy in permeability is considered with principal axis oriented at a constant but an arbitrary angle with respect to the vertical direction. The aim of the present work is to investigate the effect of the porous matrix anisotropy on the fluid flow, heat and mass transfer within the annulus using a generalised form of the momentum equation (Darcy, Brinckman, Forchheimer model).

2. PHYSICAL MODEL AND MATHEMATICAL FORMULATION

The problem under consideration is a horizontal annulus filled with an anisotropic porous medium having a constant porosity ε and saturated with an incompressible and Newtonian binary fluid as shown in Figure 1. The inner cylinder of radius R_i^* and the outer cylinder of radius R_o^* are maintained at uniform and constant temperature and concentrations T_i, C'_i and T_0, C'_0 ($T_i > T_0$ and $C'_i > C'_0$). The inner cylinder is fixed while the outer is rotating in the counter-clockwise direction with angular velocity Ω. The anisotropy is characterized by the permeabilities K_1 and K_2 along the two principal axes of the porous matrix and the orientation angle γ, defined as the angle between the vertical direction and the principal axis with permeability K_1. We assume in this analysis that the fluid has a constant thermophysical properties except for the density in buoyancy term (Boussinesq approximation).

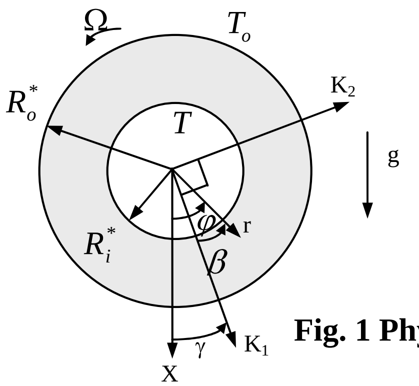


Fig. 1 Physical domain

Taking into account the above assumptions and hypothesis and using the following dimensionless parameters

$$\theta=\frac{T-T_o}{T_i-T_o}, V=\frac{v^*}{\Omega R_o^*}, U=\frac{u^*}{\Omega R_o^*}, p=\frac{p^*}{\rho_0\left(\Omega R_o^*\right)^2}\ \chi=\frac{R_i^*}{R_o^*},\ r=\frac{r^*}{R_o^*-R_i^*},\ R_i=\frac{\chi}{1-\chi},\ R_o=\frac{1}{1-\chi}\ \ C=\frac{C'-C'_0}{C'_i-C'_0} \quad (1)$$

$$Ra=\frac{g\beta\left(T_i-T_o\right)\left(R_o^*-R_i^*\right)^3}{\nu_f\,\alpha},\ \mathrm{Re}=\frac{\Omega R_o^*\left(R_o^*-R_i^*\right)}{\nu_f},\ Da=\frac{K_1}{\left(R_o^*-R_i^*\right)^2},\ \mathrm{Pr}=\frac{\nu_f}{\alpha},\ R_v=\frac{\mu_{eff}}{\mu},\ R_c=\frac{k_{eff}}{k_f}$$

The governing equations (conservation of mass, momentum, energy and species) of the problem can be written in dimensionless form as

$$\nabla\cdot\vec{V}=0 \quad (2)$$

$$\frac{1}{\varepsilon^2}\left(\vec{V}\cdot\nabla\right)\vec{V}=-\nabla p-\frac{Ra}{\mathrm{Pr}\,\mathrm{Re}^2}\left[\left(\theta+NC\right)\cos(\varphi)\vec{e}_r-\left(\theta+NC\right)\sin(\varphi)\vec{e}_\varphi\right]$$
$$+\frac{R_v}{\mathrm{Re}}\nabla^2\vec{V}+\frac{1}{\mathrm{Re}\,Da}\left[\overline{\overline{K}}\right]^{-1}\vec{V}-\frac{C_f}{\sqrt{Da}}\left[\sqrt{\overline{\overline{K}}}\right]^{-1}\vec{V}\left|\vec{V}\right| \quad (3)$$

$$V\frac{\partial\theta}{\partial r}+\frac{U}{r}\frac{\partial\theta}{\partial r}=\frac{R_c}{\mathrm{Re}\,\mathrm{Pr}}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\theta}{\partial\varphi^2}\right] \quad (4)$$

$$V\frac{\partial C}{\partial r}+\frac{U}{r}\frac{\partial C}{\partial r}=\frac{1}{\mathrm{Re}\,Le\,\mathrm{Pr}}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 C}{\partial\varphi^2}\right] \quad (5)$$

The local Nusselt number at the inner cylinder is defined by: $Nu_i=\ln\left(\frac{R_i}{R_0}\right)\left(r\frac{\partial\theta}{\partial r}\right)_i$ (6)

The average Nusselt number is given by: $\overline{Nu_i}=\frac{1}{2\pi}\int_0^{2\pi}Nu_i(\varphi)d\varphi$.

The above governing equations are to be solved with the following boundary conditions:
On the surface of the inner cylinder:
$r=R_i\quad and\quad 0<\varphi<2\pi\ :\ V=U=0$, C=1, $\theta=1$ (7)

On the surface of the outer cylinder:
$r=R_o\quad and\quad 0<\varphi<2\pi\ :\ V=0,\quad U=1$, C=0, $\theta=0$ (8)

3. NUMERICAL PROCEDURE

The well known control volume formulation based on finite difference is used to solve the governing equations of the problem with specified boundary conditions. This technique is well described by Patankar [1981]. The solution was considered to have converged to the steady state, when the absolute value of the maximum relative difference between two consecutive time steps was less than a prescribed value chosen to be 10^{-5}.

4. RESULTS AND DISCUSSION

Because of the numerous parameters which govern the problem under study, results are presented only for a permeability ratio ranging from 0.1 to 10 and the buoyancy ratio ranging from -15 to 15. All the other parameters are held fixed (Ra=10^4, Re=100, Da=10^{-2}, γ=0°, χ= 0.5, Le=1). The porosity of the porous matrix was assumed constant (ε = 0.95) and the inertia coefficient was selected to be C_f = 0.1.

4.1 VALIDATION OF THE PRESENT WORK

In order to validate the present numerical code, we have compared the predicted results with the work of Yoo [1998] in a concentric horizontal annulus without porous media (Da$\rightarrow\infty$). It can be seen from the figure 2 and figure 3 that the solution obtained with our code is in good agreement with the numerical results of Yoo [1998].

4.2 EFFECT OF THE PERMEABILITY RATIO K*

The effect of permeability ratio K* on streamlines and isotherms is shown in figure 3. The permeability ratio K* increases from the left to the right (Figure 3b represents the isotropic case). We observe the apparition of two cells in the annulus. The cell at the left side is more elongated than the right cell due to the rotational effect of the outer cylinder.
For a porous matrix having a large permeability in the horizontal direction (K*<<1), a thinner boundary layer is observed to cluster under the inner cylinder which leads to an increase in heat transfer rate (cf. Figure 4). Also a thermal plume is observed to emerge above the inner cylinder. The intensity of this thermal plume decreases when K* is augmented progressively. Particularly for K*=5, as the permeability is being small in the horizontal direction, a greater resistance is offered to the fluid in that direction by the porous matrix which results to the generation of a convection cell above the inner cylinder and as a consequence, two thermal plumes will emerge above the inner cylinder. As K* is increased to 10, these two thermal plumes tend to disappear. Figure 4 shows the mean Nusselt number as a function of permeability ratio K*. It is observed that the heat transfer is enhanced for a larger permeability in the horizontal direction. It is also observed a noticeable increase of the Nusselt number when K* varies from 2 to 0.5. Figure 5 displays mean Nusselt number as a function of buoyancy ratio for two value of K*(0.25 and 10). The mean Nusselt number predictions for the considered range of buoyancy ratio numbers are found to converge at N $=-1$. For each parameter K*, the Nusselt number predictions are lower in the opposing flow region (negative N value) than for the corresponding N value in the aiding flow (positive N value) due to the combined effect of the buoyancy forces. Moreover, higher Nusselt number predictions are achieved with higher absolute values of N as; in this case, the solutal buoyancy forces contribute to the overall diffusion rate (dominant mass transfer regime). It also observed that at a constant buoyancy ratio N, we obtain a higher Nusselt number with a porous matrix having a larger permeability in the horizontal direction.

4.3 EFFECT OF THE REYNOLDS NUMBER

The effect of Reynolds number on streamlines and isotherms is shown on figure 6 for K*=10. The Reynolds number was varied over a range from 10 to 400. The lower limit represents the natural convection regime, while the upper limit represents the forced convection regime. For lower limit the flow is mainly induced by the buoyancy forces, which are generated due to the temperature gradient between the inner and outer cylinder. The streamlines consist of one pair of cells, one on the right and one on the left portion of the annulus. We observe from the figure that when the Reynolds number is increased, the left cell become stronger than the right one. Due to the rotation of the outer cylinder in counter-clockwise direction, in the left portion of the annulus the forced convection flow is added to the natural convection flow but they oppose each other in the right portion.
For low Re number, we observe also the emergence of two thermal plumes above the inner cylinder. As Re number is increased the intensity of these two thermal plumes diminish and

will disappear at high Re number. At high Re number (Re>200), we observe that the flow becomes isothermal near the outer cylinder and that the isotherms become slightly deformed and concentric near the inner cylinder.

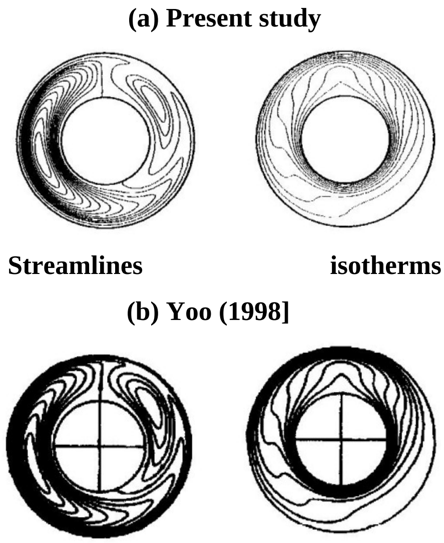


Figure 2a. Comparison of the streamlines and isotherms (Re=100, Ra=10^4 and χ=0.5)

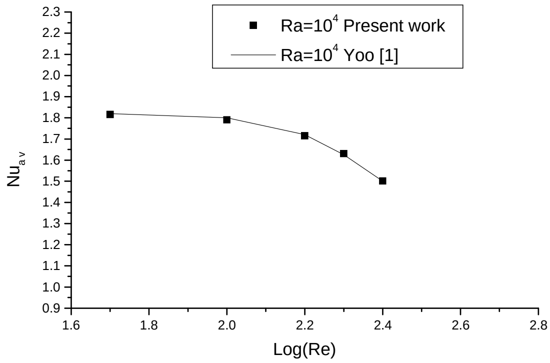
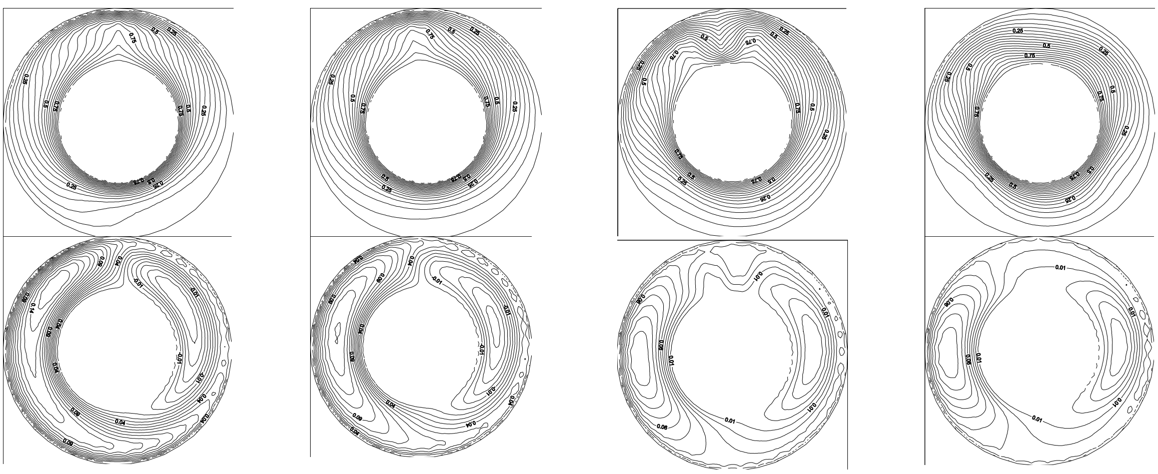


Figure 2b. Comparison of the average Nusselt Number for various Reynolds numbers (Ra=10^4 and χ=0.5)



K*=0.1 (a) K*=1 (b) K*=5 (c) K*=10 (d)

Figure 3 streamlines and isotherms for different permeability ratio K*

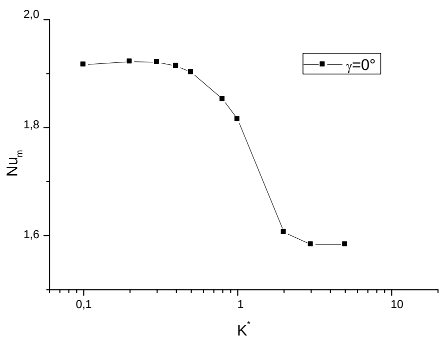


Figure 4 Mean Nusselt number vs. K*

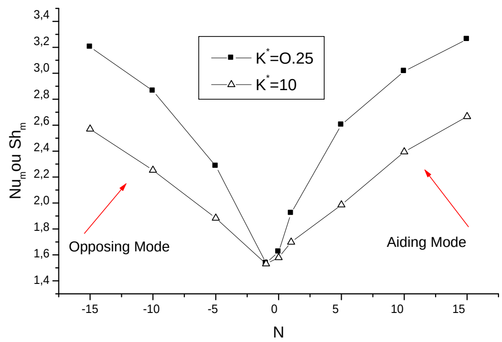
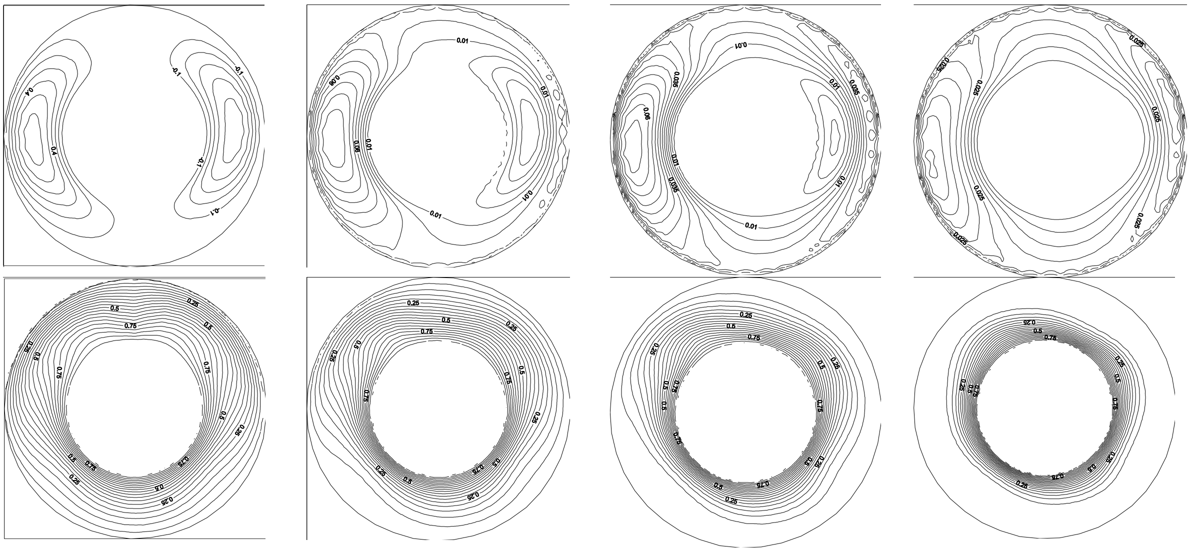


Figure 5 Mean Nusselt number vs. buoyancy ratio



Re=10 Re=100 Re=200 Re=400

Figure 6 streamlines and isotherms for different Reynolds numbers

5. CONCLUSION

The problem of thermosolutal mixed convection within a horizontal annulus filled with anisotropic matrix has been investigated numerically. Isothermal and solutal boundary conditions are applied on both the inner and outer cylinders with the inner being warmer and highly solutal and the outer rotating with constant angular velocity in the counter clockwise direction. The porous medium is assumed to be hydrodynamically anisotropic with the principal axes of anisotropic permeability inclined with respect to the gravity force. The generalized model of Darcy, Brinckman and Forchheimer is used in the formulation. The present results showed that the permeability ratio has a significant effect on the flow patterns, heat and mass transfer characteristics.

References

[1] Aboubi, K., Robillard, L., Vasseur, P. [1998], Natural convection in horizontal annulus filled with an anisotropic porous medium, *Int. J. of Num. Meth. Heat and Fluid flow*, vol. 8, pp 689-702.

[2] A.M. Al-Amiri, K.M. Khanafer (2006) “Numerical simulation of double-diffusive mixed convection within a rotating horizontal annulus” International Journal of Thermal Sciences 45: 567–578

[3] Khanafer, K. Chamkha, A. [2003], Mixed convection within a porous heat generating horizontal annulus, *Int. J . Heat Mass Transfer*, Vol.46, pp. 1725-1735.

[4] Patankar, S.V. [1980], *Numerical heat transfer and fluid flow*, Mc Graw Hill Book Company.

[5] Teamah, M. (2007) “Numerical simulation of double-diffusive laminar mixed convection in a horizontal annulus with hot, solutal and rotating inner cylinder” International Journal of Thermal Sciences 46: 637–648

[6] Vafai, K. Tien, C.L. [1981], Boundary and inertia effects on flow and heat transfer in porous media, *Int. J. Heat Mass Transfer*, vol. 24, pp 195-203.

[7] Yoo, J.S. [1998], “Mixed convection of air between two horizontal concentric cylinders with a cooled rotating outer cylinder, *Int. J. Heat Mass Transfer*, Vol.41, pp. 293-302.