# EEFECT OF THE MAGNETIC FIELD INCLINATION ON ENTROPY GENERATION IN A GRAVITY-DRIVEN LIQUID FILM ALONG AN INCLINED HEATED PLATE

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## ABSTRACT

The purpose of this work is to investigate the entropy generation in a conducting fluid flowing along an inclined heated plate in the presence of a magnetic field. The upper surface of the liquid film is considered free and adiabatic. Velocity and temperature profiles are obtained and used to compute the entropy generation number profiles. The effects of the applied magnetic field and its inclination on entropy generation are examined.
**Keywords**: Entropy, Inclined plate, Liquid film, Magnetic field, Thermodynamic analysis.

## NOMENCLATURE

| | |
|---|---|
| $a$ | thermal diffusivity ($m^2.s^{-1}$) |
| $\vec{B}$ | magnetic field (T) |
| $Br$ | Brinkman number |
| $C_P$ | specific heat ($J.kg^{-1}.K^{-1}$) |
| $g$ | gravitational acceleration ($m.s^{-2}$) |
| $Pe$ | Peclet number |
| $q$ | wall heat flux ($W.m^{-2}$) |
| $S_G$ | entropy generation rate ($W.m^{-3}.K^{-1}$) |
| $T$ | temperature (K) |
| $u$ | axial velocity ($m.s^{-1}$) |
| $U$ | dimensionless axial velocity |
| $x$ | axial distance (m) |
| $X$ | dimensionless axial distance |
| $y$ | transverse distance (m) |
| $Y$ | dimensionless transverse distance |

Greek symbols

| | |
|---|---|
| $\alpha$ | magnetic field inclination (rad) |
| $\delta$ | liquid film thickness (m) |
| $\Delta T$ | reference temperature difference |
| $\mu$ | dynamic viscosity ($kg.m^{-1}.s^{-1}$) |
| $\lambda$ | thermal conductivity ($W.m^{-1}.K^{-1}$) |
| $\theta$ | inclination angle of the plate (rad) |
| $\Theta$ | dimensionless temperature |
| $\Omega$ | dimensionless temperature difference, $\Delta T/T_0$ |
| $\rho$ | density of the fluid ($kg.m^{-3}$) |

Subscripts

| | |
|---|---|
| $b$ | bulk value |
| $m$ | maximum value |
| $0$ | reference value |

## 1. INTRODUCTION

Entropy generation is closely associated with thermodynamic irreversibility, which is encountered in all heat transfer processes. Different sources are responsible for generation of entropy such as heat transfer and viscous dissipation [1, 2]. The analysis of entropy generation rate in a circular duct with imposed heat flux at the wall and its extension to determine the optimum Reynolds number as function of the Prandtl number and the duty parameter were presented by Bejan [2, 3]. Sahin [4] introduced the second law analysis to a viscous fluid in circular duct with isothermal boundary conditions.

In another paper, Sahin [5] presented the effect of variable viscosity on entropy generation rate for heated circular duct. A comparative study of entropy generation rate inside duct of different shapes and the determination of the optimum duct shape subjected to isothermal boundary condition were done by Sahin [6]. Narusawa [7] gave an analytical and numerical analysis of the second law for flow and heat transfer inside a rectangular duct. In a more recent paper, Mahmud and Fraser [8, 9] applied the second law analysis to fundamental convective heat transfer problems and to non-Newtonian fluid flow through channel made of two parallel plates. The study of entropy generation in a falling liquid film along an inclined heated plate was carried out by Saouli and Aïboud-Saouli [10]. As far as the effect of a magnetic field on the entropy generation is concerned, Mahmud et al. [11] studied the case of mixed convection in a channel.

The purpose of this article is to analyze the effect of the inclination of the magnetic field on the entropy generation in a fully developed liquid film flowing along an inclined heated plate. Expressions for dimensionless velocity and temperature, entropy generation number obtained.

## 2. PROBLEM FORMULATION AND ANALYSIS

The problem concerns a fully developed Newtonian, laminar, liquid film flowing along an inclined heated plate in the presence of a transverse uniform magnetic field $\vec{B}$ having an inclination $\alpha$ with the axial axis. The magnetic Reynolds number is assumed to be small, so that the induced magnetic field is neglected and the Hall effect of magnetohydrodynamics is ignored. Neglecting the inertia terms in the momentum equation compared to the body force and the magnetic term, the momentum equation is then:

$$\mu \frac{\partial^2 u(y)}{\partial y^2} - \sigma B^2 u^2(y) \sin^2\alpha + \rho g \sin\theta = 0 \tag{1}$$

subjected to the following boundary conditions:

No-slip condition $\quad u(0) = 0 \quad$ (2a)

Free surface $\quad \frac{\partial u(\delta)}{\partial y} = 0 \quad$ (2b)

The velocity profile is obtained by integrating Equation 1 and using the boundary conditions given by Equation 2. It may be written:

$$u(y) = \frac{\rho g \sin\theta}{\sigma B^2 \sin^2\alpha}\left(1 - \frac{\cosh\left(B\sin\alpha\sqrt{\frac{\sigma}{\mu}}(\delta - y)\right)}{\cosh\left(B\sin\alpha\,\delta\sqrt{\frac{\sigma}{\mu}}\right)}\right) \tag{3}$$

Introducing the following dimensionless variables for the velocity and the transverse distance $U(Y) = \frac{u(y)}{u_m}$, $Y = \frac{y}{\delta}$, the dimensionless velocity becomes:

$$U(Y) = \frac{\cosh(Ha\sin\alpha) - \cosh(Ha\sin\alpha(1-Y))}{\cosh(Ha\sin\alpha) - 1} \tag{4}$$

where $Ha$ is the Hartmann number defined as:

$$Ha = B\delta\sqrt{\frac{\sigma}{\mu}} \quad \text{and} \quad u_m = \frac{\rho g \sin\theta}{\sigma B^2 \sin^2\alpha}\left(\frac{\cosh(Ha\sin\alpha) - 1}{\cosh(Ha\sin\alpha)}\right) \tag{5}$$

Using the following dimensionless variables:

$$X = \frac{ax}{u_m \delta^2},\; Y = \frac{y}{\delta},\; U(Y) = \frac{u(y)}{u_m},\; \Theta(X,Y) = \frac{T(x,y) - T_0}{\Delta T} \tag{6}$$

where $\Delta T$ is a reference temperature difference defined as:

$$\Delta T = \frac{q\delta}{\lambda} \tag{7}$$

The energy equation can be written in the following dimensionless form:

$$U(Y)\frac{\partial\Theta(X,Y)}{\partial X}=\frac{\partial^2\Theta(X,Y)}{\partial Y^2}+BrHa^2\sin^2\alpha U^2(Y) \tag{8}$$

subjected to the following boundary conditions:

$$\Theta(0,Y)=0,\ \frac{\partial\Theta(X,0)}{\partial Y}=-1,\ \frac{\partial\Theta(X,1)}{\partial Y}=0 \tag{9}$$

$$\begin{aligned}\Theta(X,Y)=C_1X+\frac{\alpha}{\cosh(Ha')-1}\left[\frac{Y^2}{2}\cosh(Ha')-\frac{\cosh(Ha'(1-Y))}{Ha'^2}\right]-\frac{BrHa'^2}{(\cosh(Ha')-1)^2}\Bigg[\frac{Y^2}{2}\cosh^2(Ha')\\ -\frac{2}{Ha^2}\cosh(Ha')\cosh(Ha'(1-Y))+\frac{1}{8Ha'^2}\cosh(2Ha'(1-Y))+\frac{Y^2}{2}\ \Bigg]+C_2Y+C\end{aligned} \tag{10}$$

where $C_2$ and $C$ are constants of integration.

Using the boundary conditions (9), it is found that:

$$C_1=\frac{A_3-A_4}{A_1-A_2},\ C_2=\frac{A_1A_4-A_2A_3}{A_1-A_2} \tag{11}$$

In the above expression $A_1$, $A_2$, $B_1$ and $B_2$ can be defined by:

$$\begin{aligned}A_1=\frac{\sinh(Ha')}{Ha'(\cosh(Ha')-1)},\ A_2=\frac{\cosh(Ha')}{\cosh(Ha')-1},\ A_3=\frac{BrHa'^2}{(\cosh(Ha')-1)^2}\left(\frac{2}{Ha}\cosh(Ha')\sinh(Ha')-\frac{1}{4Ha'}\sinh(2Ha')\right)-1\\ A_4=\frac{BrHa'^2}{(\cosh(Ha')-1)^2}\left(\cosh^2(Ha')+\frac{1}{2}\right)\end{aligned} \tag{12}$$

The boundary conditions defined by Equation 9 leads the following condition on the bulk mean temperature:

$$\Theta_b(0)=0 \tag{13}$$

Substituting Equation 10 in Equation 13 and using Equation 9, the constant of integration is then:

$$\begin{aligned}C=\frac{\alpha}{(\cosh(Ha')-1)}\left[\frac{\sinh(Ha')}{Ha'^3}\right]-\frac{BrHa'^2}{(\cosh(Ha')-1)^2}\Bigg[\ \frac{2}{Ha'^3}\cosh(Ha')\sinh(Ha')-\\ \frac{1}{16Ha'^3}\sinh(2Ha')\ \Bigg]-\frac{\alpha}{6(\cosh(Ha')-1)}+\frac{BrHa'^2}{(\cosh(Ha')-1)^2}\left[\ \frac{1}{6}\cosh^2(Ha')+\frac{1}{12}\ \right]\end{aligned} \tag{14}$$

where $Ha'=B\delta\sqrt{\frac{\sigma}{\mu}}\sin\alpha$

According to Woods [13], the entropy generation rate for the present case is:

$$S_G=\frac{\lambda}{T_0^2}\left[\left(\frac{\partial T(x,y)}{\partial x}\right)^2+\left(\frac{\partial T(x,y)}{\partial y}\right)^2\right]+\frac{\mu}{T_0}\left(\frac{\partial u(y)}{\partial y}\right)^2+\frac{\sigma B^2}{T_0}u^2(y) \tag{15}$$

The dimensionless entropy generation number is defined by the following relationship:

$$N_S=\frac{\lambda T_0^2}{q^2}S_G \tag{16}$$

using the dimensionless velocity and temperature, Equation 22 can be rewritten as:

$$N_S=\frac{1}{Pe^2}\left(\frac{\partial\Theta(X,Y)}{\partial X}\right)^2+\left(\frac{\partial\Theta(X,Y)}{\partial Y}\right)^2+\frac{Br}{\Omega}\left(\frac{\partial U(Y)}{\partial Y}\right)^2+\frac{BrHa^2}{\Omega}U^2(Y) \tag{17}$$

$$N_S=N_C+N+_Y N_F+N_B \tag{18}$$

where $Pe$ and $\Omega$ are respectively the Peclet number and the dimensionless temperature difference. $N_C$ and $N_Y$, are respectively the entropy generation numbers due to the conductive heat in the axial and the transverse directions. $N_F$ is the entropy generation number due to the fluid friction and $N_B$ is the entropy generation due to the magnetic effect.

## 3. RESULTS AND DISCUSSION

The effect of the Hartmann number $Ha$ on the spatial distribution of the entropy generation number is plotted in Figure 1. As the Hartmann number increases the entropy generation number increases in the transverse direction and the apparition of minimums near the heated plate becomes clear. The entropy generation number does not become nil at the upper surface of the liquid film although it is free and adiabatic. This is because the application of the magnetic field creates additional entropy at the upper surface.

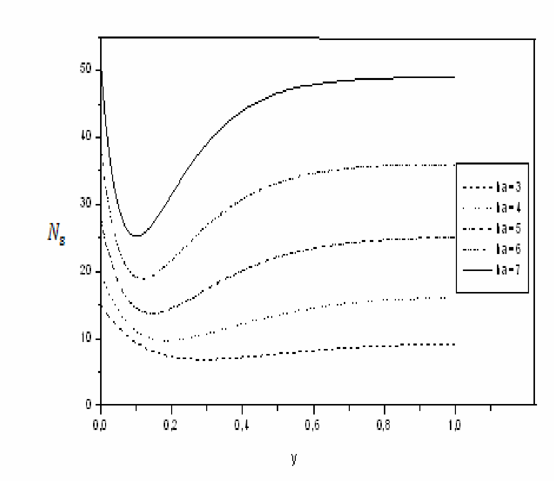


Figure 1. Effect of the Hartman number.

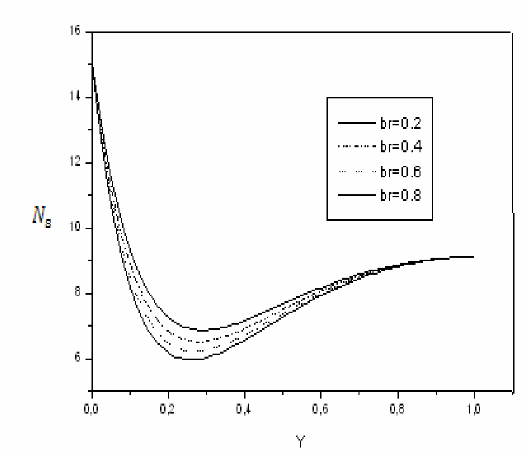


Figure 2. Effect of the Brinkman number.

Figure 2 illustrates the effect of the Brinkman number $Br$ on the spatial distribution of the entropy generation number. For a given transverse position, the entropy generation number is higher for higher Brinkman number. In all cases the heated plate acts as a strong concentrator of irreversibility.

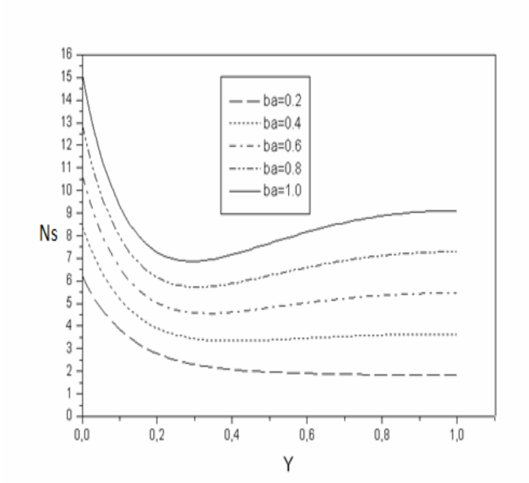


Figure 3. Effect of the dimensionless group.

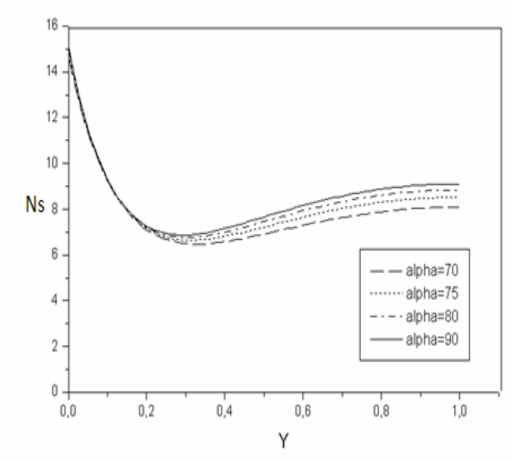


Figure 4. Effect of the magnetic field inclination.

The effect of the group parameters $Br\Omega^{-1}$ on the spatial entropy generation number is depicted in Figure 3. An inspection on this plot reveals that the entropy generation number presents minimums near the heated plate. For a given transverse position, the entropy generation number is higher for higher group parameter. The effect of the magnetic field inclination on the entropy generation number is illustrated in figure 4. As it can be seen, the higher the inclination of the magnetic field, the higher is the entropy generation number.

## 4. CONCLUSION

This paper presents the application of the second law analysis of thermodynamics to a falling liquid film along an inclined heated plate in the presence of an inclined magnetic field. The effects of the Hartmann number, Brinkman number, the group parameter and the inclination of the magnetic field entropy generation number are discussed.

From the results the following conclusions could be drawn:

a) The entropy generation number increases as Brinkman number increases.
b) The entropy generation number increases as Hartmann or the group parameter increases.
c) The entropy generation number increases as the magnetic field incraeses.

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