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THERMAL GRADIENT DRIVEN FLOWS IN MICROCHANNEL

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RESUME
On étudie la transpiration thermique stationnaire dans un très large micro conduit rectangulaire. Une solution analytique de type continu est proposée pour des nombres de Knudsen inférieurs à 0.25. La comparaison avec les résultats d'autres approches est satisfaisante.

1. INTRODUCTION
Rarefied gas flows may be generated by means of tangential temperature gradients along the channel walls: then the fluid starts creeping from the cold towards the hot regions. Since, Reynolds introduces in 1879 the term "thermal transpiration", some authors have analyzed thermal creep flows through kinetic approaches ([1] [2]) [3]). The gas velocity, the heat flux profiles and the mass fluxes have so been numerically obtained over a large Knudsen number range. Velocity and temperature profiles have also been previously derived from the Navier-Stokes equations, assuming only small compressible effects [4].
We analyze here the thermal creep phenomenon in rectangular micro channels for fully compressible gases. The temperature of the walls varies as a function of the x stream wise coordinate. The pressures in the tank are maintained equal: thus we study the stationary flows in slip regime. In order to obtain an analytical modelling we estimate the magnitude of the non-dimensional macroscopic parameters and non-dimensional gradients. Analytical expressions of temperature, velocity, pressure distributions, mass flow rate and heat flux are derived from the full compressible NS equations applying a perturbation method. Numerical and analytical calculations are carried out for different micro channels sizes, wall temperature gradients, pressures and gases. The Knudsen numbers based on the channel height, vary from 0.01 to 0.25. The comparisons between analytical and numerical results confirm the validity of the analytical approach. A comparison with the results of other authors applying the kinetic model equations is also discussed.

2. CONTINUUM EQUATIONS AND SLIP BOUNDARY CONDITIONS
2.1 Continuum equations
We consider the flow in rectangular micro channels connecting two reservoirs maintained at the same pressure. The temperature in inlet and outlet reservoirs are fixed, respectively equal to T_{in} and to T_{out}. The temperature along the wall is assumed to vary as a linear function of the x stream wise coordinate. Then the gas flows from the cold to the hot tank, according to the thermal creep effect. In this study we analyze this thermal phenomenon in channel where the value of the height H is bigger than that of the mean free path λ and much smaller than that of the channel length L or width w: this configuration is considered as two infinite parallel plates and the physical parameters depend only on two (x, y) spatial coordinates. The NS equations are used to describe the stationary flows and the viscosity is treated within the variable hard sphere (VHS) model [5].

2.2 Slip boundary conditions

The boundary conditions for the NS equations system in slip regime are the symmetry conditions on the axis and the slip velocity and the temperature jump conditions on the wall. Considering the boundary condition of the first order in Knudsen number, the complete slip boundary condition on the upper solid wall reads [6][7] :

$$u_s = -\sigma_p \frac{\mu}{p} v_m \left(\frac{\partial u}{\partial y} \right)_w + \sigma_T \frac{\mu}{\rho} v_m \left(\frac{\partial \ln T}{\partial x} \right)_w \qquad (1)$$

$v_m = \sqrt{2RT_w(x)}$ is the most probable molecular velocity at the surface temperature T_w, σ_p is the slip velocity coefficient and σ_T the thermal slip coefficient. In this article we use the velocity slip coefficient $\sigma_p^K = 1.012$ given in [6], under the full accommodation assumption of the molecules at the wall, the well known value of σ_T (equal to 0.84) suggested also in [6], the calculated helium σ_T value from [7], and also the measured air σ_T value from [8].Finally a boundary condition proposed by [6] is chosen to describe the temperature jump at the wall. Furthermore the mean free path is written as a function of macroscopic parameters

$$\lambda = k_\lambda \frac{\mu}{p} \sqrt{RT} \quad , \qquad k_\lambda = A(\omega) = \frac{2(7-2\omega)(5-2\omega)}{15\sqrt{2\pi}} \qquad (2)$$

The coefficient k_λ depends on the molecular interaction model: we use, the expression deduced in [5] for the variable hard sphere model (VHS) more general than the HS model, where the coefficient $A(\omega)$ depends only on the gas via ω ($0.5 \le \omega \le 1$) the viscosity index.

3. ESTIMATION OF VARIOUS TERM MAGNITUDE

Steady flows are studied now, therefore in the following analysis the time derivatives vanish. The variables are non-dimensioned in the following manner: the stream wise coordinate x by the channel length L, and the wall normal coordinate y by the channel height H. The channel height-to-length ratio ε will be small compared to unity: $\varepsilon = H/L << 1$.The stream wise velocity u is normalized using the velocity u_R that is close to the mean velocity at the channel exit and of the same order as the characteristic velocity of the mass flow rate. Then the velocity v is normalized by εu_R. The pressure is normalized by using the pressure value at the channel exit p_{out} (the subscript "out" refers to outlet conditions). The temperature is normalized using the value of the wall temperature fixed $T_{w_{out}}$ at the channel outlet. Finally the density is normalized by using the appropriate outlet value $\rho_{w_{out}}$, and the viscosity coefficient by the value $\mu_{w_{out}}$ corresponding to the outlet wall temperature $T_{w_{out}}$.The so defined non dimensional parameters are underlined. We now introduce a second type of non-dimensional variables: $\tilde{\theta}$ referring to the temperature and $\tilde{\Pi}$ to represent the pressure, both to be used in the dimensional analysis of the equation system:

$$\tilde{\theta} = \frac{T - T_{axe}(x)}{T_w(x) - T_{axe}(x)}, \; \tilde{\Pi} = \frac{p - p_{out}}{p_M - p_{out}}, \; \frac{\partial \tilde{\Pi}}{\partial \tilde{x}} = \frac{\partial}{\partial \tilde{x}} \left(\frac{p - p_{out}}{p_M - p_{out}} \right) = \frac{1}{p_M - p_{out}} \frac{\partial p}{\partial \tilde{x}} = \frac{p_{out}}{\Delta p_M} \frac{\partial \tilde{p}}{\partial \tilde{x}} \qquad (3)$$

T_{axe} is the temperature on the symmetry axis. This second form of non-dimensional temperature is chosen in order to obtain $\tilde{\theta}$ and the derivative of $\tilde{\theta}$ along the y-axis, both of zero order. Furthermore, in the flows under consideration, the pressure keeps anywhere a value close to p_{out} (reservoir value). Therefore, like for the temperature and for similar reasons, we introduced a $\tilde{\Pi}$ form of pressure profile, where p_M represents the maximum pressure value in the channel. Thus, we obtain derivatives of $\tilde{\Pi}$ along the x-axis of same order as $\tilde{\Pi}$ itself and the derivative in the y-direction would have the same form. Defining the Knudsen, Reynolds and Mach numbers based on the normalizing parameters, we obtain:

$$\mathrm{Re} = \frac{\rho_{w_{out}} u_R H}{\mu_{w_{out}}}, \qquad Ma = \frac{u_R}{a_{w_{out}}}, \qquad Kn = \frac{\lambda}{H} = k_\lambda \sqrt{\gamma}\,\frac{Ma}{\mathrm{Re}}, \quad a_{w_{out}} = \sqrt{\gamma R T_{w_{out}}} \tag{4}$$

It is necessary to remember here that we have restricted our study to the slip flow regimes characterized by a Knudsen number below 0.3. Then, owing to the boundary conditions of the system, the flows under consideration will be rather low speed flows and that the typical values of their Reynolds number will be of zero or ε order ($O(\varepsilon)$,O(*1*)).

4. THERMAL BOUNDARY CONDITIONS, ANALYTICAL EXPRESSIONS

The different steps to obtain the analytical approximate equation system are the following.

--In the NS system we use the general expression linking Kn, Re and Ma numbers (4).

--Assuming that in usual micro channel flows Re ~1 or $\mathrm{Re} \sim \varepsilon$ we can eliminate some non-dimensional terms.

--Finally we estimate the order of magnitude of the remaining terms using energy balances in the channel: first a thermal balance concerning the gas macroscopically in the rest; then a balance involving the real gas flow.

Then, two main features are utilized to simplify the system even more:1)we specify our choice concerning the "driver" inlet/outlet temperature difference noting $T_{out} - T_{in} \sim T_{out} \sim T_{in}$, or $\tilde{T} \sim 1$ $\partial\tilde{T}/\partial\tilde{x} \sim 1$, 2) we estimate the magnitude orders of the thermal quantities characterizing respectively the transversal transfer and the stream wise transfer, assuming that the fluid motion does not change the relative magnitude order of these various transfers. This assumption is based on the very small magnitude of the mass flow rate under consideration. Thus we obtain u_R, close to the outlet velocity ($u_R \cong u_{out}$), that we use as characteristic value to normalize the two velocity components u and v. An explicit Re expression is deduced: $u_R = \dfrac{\varepsilon}{2p_{out}}\dfrac{R}{\mathrm{Pr}}\mu_{w_{out}}\dfrac{T_{w_{out}}}{L}$, $\quad \mathrm{Re} = \dfrac{\rho_{w_{out}} u_R H}{\mu_{w_{out}}} = \dfrac{\varepsilon}{2\,\mathrm{Pr}}$, where Pr is the Prandtl number. Following the way described above, we obtain the analytical transversal velocity profile:

$$\tilde{u}(\tilde{y}) = \left(\frac{3}{2}\left(4\tilde{y}^2 - 1\right) - 6Kn_*\tilde{T}^{\omega+0.5}\right)\frac{2\sigma_T K_w \,\mathrm{Pr}\,\tilde{T}^{\omega} - \tilde{T}\tilde{Q}}{1 + 6Kn_*\tilde{T}^{\omega+0.5}} + 2\sigma_T K_w \,\mathrm{Pr}\,\tilde{T}^{\omega} \tag{5}$$

The notations used are: $Kn_* = K_{slip} Kn_{out}$, $\quad \dfrac{\partial\tilde{\mathrm{T}}}{\partial\tilde{\mathrm{x}}} = K_w$, $\quad \mathrm{K_{slip}} = \sigma_p \dfrac{\sqrt{2}}{k_\lambda}$ and $\tilde{Q}$ is a non-dimensional mass flow rate, defined as $Q = \tilde{Q}\mu_{out} H / 2\,\mathrm{Pr}\,L$. The pressure distribution is obtained via $\tilde{\Pi}$, the second non-dimensional pressure form:

$$\tilde{\Pi} = \tilde{\Pi}_A - \tilde{Q}\,\tilde{\Pi}_B, \quad \tilde{\Pi}_A = \frac{2\sigma_T \,\mathrm{Pr}}{Kn_*}\left(D_{\omega+0.5}(\tilde{x}) + \frac{1}{6Kn_*(\omega+0.5)}\ln\frac{1 + 6Kn_*\tilde{T}_{w_{in}}^{\omega+0.5}}{1 + 6Kn_*\tilde{T}^{\omega+0.5}}\right), \tag{6}$$

where D_υ is defined as $D_\upsilon(\tilde{x}) = \left(\tilde{T}^\upsilon(\tilde{x}) - \tilde{T}^\upsilon_{w_{in}}\right)/\upsilon$. Then noting $c = \dfrac{6}{1+3Kn_*}$., we obtain

$$\tilde{\Pi}_B = \frac{c}{K_w}\left(\left(1+\frac{1}{2}cKn_* + \frac{1}{4}c^2Kn_*^2\right)D_{\omega+2}(\tilde{x}) - cKn_*\left(1+cKn_*\right)D_{2\omega+2.5}(\tilde{x}) + c^2Kn_*^2D_{3\omega+3}(\tilde{x})\right),$$

Then using $\tilde{\Pi}(1)=1$ in the outlet section and writing $D_\upsilon(\tilde{x})\big|_{\tilde{x}=1} = D_\upsilon$, we obtain:

$$\tilde{Q} = \frac{2\sigma_T \Pr}{Kn_*}\left(D_{\omega+0.5} + \frac{1}{6Kn_*(\omega+0.5)}\ln\frac{1+6Kn_*\tilde{T}^{\omega+0.5}_{w_{in}}}{1+6Kn_*\tilde{T}^{\omega+0.5}}\right) / \left(\frac{c}{K_w}\left(\left(1+\frac{1}{2}cKn_* + \frac{1}{4}c^2Kn_*^2\right)D_{\omega+2}(\tilde{x}) - cKn_*\left(1+cKn_*\right)D_{2\omega+2.5}(\tilde{x}) + c^2Kn_*^2D_{3\omega+3}(\tilde{x})\right)\right), \tag{7}$$

5. NUMERICAL SIMULATION AND COMPARISON WITH THE ANALYTICAL APPROACH AND OTHER THEORIES

5.1 Numerical modelling

For the numerical simulation we solve the full NS equation system. The computations are carried out for a two dimensional flow between the symmetry axis and the parallel plate at distance $H/2$. As in the previous sections, the flow is sustained by the temperature gradient. The condition of equal pressures in the reservoirs is applied as equal pressures at the channel ends.. The other flow parameters at the inlet and outlet sections of the channel are deduced from the characteristic method. The computations are performed for different gases. The calculations are carried out for a channel of $H = 10\mu m$ in height and $L = 1cm$ in length. We compared here analytical and numerical profiles of the flow parameters in the channel notably under two different types of flow conditions:

--The temperature in the inlet tank is equal to 295K. Various values of the pressure (the same in the two tanks) are considered (see Table1), which correspond to different values of the Knudsen number. The influence of the Knudsen number on the flow is studied. Nitrogen flow only is considered here

--The pressure in both tanks equals the atmospheric pressure p_{atm}, the temperature in the inlet tank equals 295K. Various values of the temperature difference between the tanks are considered (Table2) and its influence on the flow is studied. Nitrogen flow only is considered

5.2 Comparisons and comments.

The temperature difference between the two reservoirs is maintained and, contrarily to that occurred in classical unsteady experiments, the pressures are kept equal and constant. Thus, we obtain a steady flow from the cold to the hot reservoir, while a non-linear pressure profile forms along the channel and of course the mass flow rate does not vanish. The mass flow rates through the channel calculated numerically and analytically are compared in Tables 1 and 2 for different flow conditions. The analytical and numerical values of mass flow rate coincide practically for the small Knudsen numbers and start to differ from about 3% for Knudsen numbers greater than 0.25. The comparison with the results obtained in [2] applying the S-model are also carried out. The difference in the mass flow rate between the proposed analytical approach and the method used in [2] is about $5-7\%$. This difference may be explained considering the different interaction molecular models used in the two approaches: respectively, the hard sphere model in [2], and the VHS model here. Note that the author of

[2] models the collision integral of BE using S-model which gives a correct Prandtl number only for monatomic gases. The influence of the temperature gradient along the wall is studied in Table2: the mass flow rate increases with the stream wise temperature gradients.

Table 1

Dimensional mass flow rate for the nitrogen flow obtained using**:** present NS numerical approach(NS num.sol), present analytical approach (NS anal.sol.), numerical solution of the kinetic equation (S-model) [2]. The inlet and outlet pressures are equal to the atmospheric pressure. The temperature in the inlet reservoir equals 295K, $\sigma_T = 0.923$ [8]. The temperature difference between the inlet and outlet sections equals 300K

p_{out}	p_{atm}	$0.1p_{atm}$	$0.05p_{atm}$
Kn_{out}	0.0126	0.126	0.253
QNS (num.sol.)$\times 10^{-12}$ *kg/s*	2.929	2.940	2.943
QNS (anal.sol.)$\times 10^{-12}$ *kg/s*	2.929	2.910	2.851
QBGK(S-model) $\times 10^{-12}$ *kg/s*	2.929	2.927	2.404

Table 2

Dimensional mass flow rate for the nitrogen flow obtained using: present NS numerical approach (NS num.sol.)), present analytical approach (NS anal.sol.), numerical solution of the kinetic equation (S-model) [2]., $\sigma_T = 0.923$ [8]. The temperature in the inlet reservoir is 295K Pressure reservoir is 1 atm.

$T_{out} - T_{in}$	140	300	600
Kn_{out}	0.008	0.0126	0.0209
QNS (num.sol.)$\times 10^{-12}$ *kg/s*	1.449	2.929	5.371
QNS (anal.sol.)$\times 10^{-12}$ *kg/s*	1.449	2.929	5.371
QBGK(S-model)$\times 10^{-12}$ *kg/s*	1.449	2.929	5.596

The influence of the reservoir pressures (*i.e.* of the Knudsen number) is studied in Table1. The dimensional mass flow rate depends weakly on the reservoir pressure for the pressure range under consideration, which is not surprising considering analytical expressions (7). Moreover note that here the pressure increases when the Knudsen number increases.

Then it is interesting to note a general feature concerning the velocity profiles: in the first half of the channel the maximum of the velocity is found on the wall and the gas flows increase the pressure, while in the second part the maximal velocity is on the axis, the velocity profile has a typical form similar to that of the Poiseuille flow while the gas pressure decreases along the stream wise direction (see in figs1and 2 the case of He). Moreover when the Knudsen number increases the profiles become smoother and finally quasi-uniform. Generally, the analytical and numerical profiles are in a very good agreement. Finally, when comparing our results with a kinetic approach [2] we obtain a reasonable agreement, if we exclude a part of the transversal velocity profiles: near the wall the kinetic and continuum profiles are different. This local difference may be partially explained by the Knudsen layer effects.

6. CONCLUDING REMARKS

Globally, our analytical profiles agree perfectly with the "exact" numerical results and they agree reasonably with the results of the numerical solution of the S-model kinetic equation if we disregard the velocity profiles in the closest neighbouring of the wall, where the discrepancies are more important. Such analytical expressions are useful in the development. of various micro devices using the thermal creep effect to start and maintain the gas motion.

The main noticeable physical features appearing from the previous analytical description of the thermally creeping flows, may be summarized as follows: 1) the mass flow rate depends

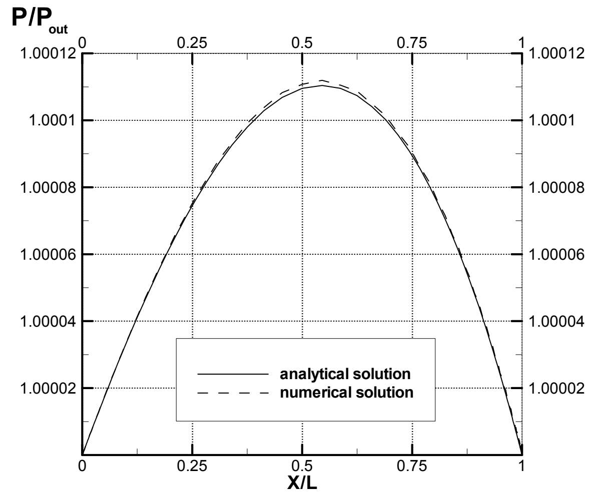

Figure 1 Pressure profiles for helium flow, $Kn_{out} = 0.036$. The solid curve is the analytical solution, the dashed line represent the numerical solution

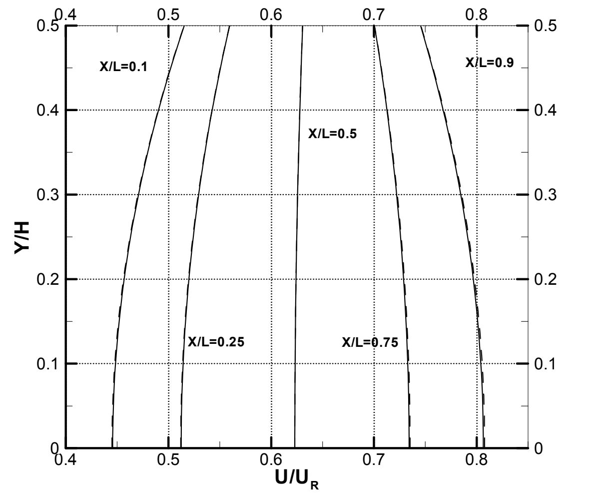

Figure2 Velocity profiles for helium flow, $Kn_{out} = 0.036$.The solid curve is the analytical solution, the dashed lines represent the numerical solution

weakly on the reservoir pressure and increases when the stream wise temperature gradient increases, 2) the pressure variation along the stream wise direction is of second order magnitude according to the Knudsen number and it is the same for the streamwise dimensionless pressure gradient. This variation has a non linear form according to x and its maximal value in reached in a fixed point x close to the middle of channel, 3) the curvature of the velocity transversal profiles changes also at this point. In the second part of the channel this curvature becomes similar to that of isothermal profiles while it was opposite in the first part; but the weight of the slip velocity remains everywhere more important than in the isothermal case, 4) the non dimensional transversal heat flux is a second order quantity according to the aspect ratio, as do the relative transversal temperature differences. These parameters depend sensitively on the thermal slip coefficient σ_T.

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