EFFECT OF A PERIODIC TEMPERATURE ON DOUBLE DIFFUSIVE CONVECTION IN A POROUS SQUARE ENCLOSURE SUBMITTED TO CROSS GRADIENTS OF TEMPERATURE AND CONCENTRATION

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ABSTRACT

Double diffusive natural convection in a square porous cavity submitted to cross gradients of temperature and concentration, is investigated numerically. The temperature of the lower horizontal surface (hot temperature) is varied sinusoidally with time, while that of the opposite surface (cold temperature) is maintained constant. The vertical walls are considered at constant but different concentrations. The parameters of the study are the amplitude of the variable temperature ($0 \le a \le 1$) and its period ($0.01 \le \tau \le 1$). The values of the thermal Rayleigh number, the ratio of the buoyancy forces and the Lewis number were fixed at 100, 0.1 and 10 respectively. The obtained results show that the heat transfer, the mass transfer and the flow intensity could be significantly enhanced, with respect to the case of a constant heating temperature, by a proper choice of the parameters related to the periodic temperature.

1. INTRODUCTION

Thermosolutal natural convection induced in porous media saturated by binary mixtures represents an extremely interesting subject due to its presence in many practical engineering applications such as the transport of contaminants in saturated soils, the electrochemical process, the migration of nuclear waste in the soil and drying process. In addition, the development of theoretical models, numerical algorithms and experimental approaches constitutes a solid basis for the advancement of knowledge in this field. However, a literature review shows that most of the previous works considered the case of rectangular cavities subjected either to horizontal [1-2] or vertical [3-4] temperature and concentration gradients for different types of boundary conditions (constant temperatures and concentrations or uniform prescribed heat and mass fluxes). Comparatively, few works have been reported on rectangular cavities filled with a porous medium subjected to cross thermal and concentration gradients [5-6]. In spite of this fact, it was obtained that boundary conditions consisting of cross temperature and concentration gradients lead to important and varied results such as the multiplicity of solutions, oscillating phenomena or complex flow structures and therefore the problem deserves to be reconsidered in order to investigate aspects not yet addressed. On the other hand, the thermal boundary conditions imposed in these studies are constant temperatures or heat fluxes. However, in many practical applications, the energy provided to the system is variable in time and gives rise to unsteady natural convection flows. The interest of time variable thermal boundary conditions is shown in the works conducted for unsteady natural convection in the case of square cavities [7-8]. Studies related to the effect of temperature modulation on the thermosolutal convection in a fluid saturated porous medium have received less attention. Among the available works, we can mention those of Antohe and Lage [9-10] related to porous enclosures subjected to intermittent heating from the side.

Yet, to the best knowledge of the authors, there appears to be no studies in the literature concerned with double diffusive convection in a porous medium heated with a periodically varying temperature and submitted to cross gradients of temperature and concentration. The object of the present study is motivated by the lack of such studies. The study is conducted numerically and the effect of the parameters related to the variable heating temperature (amplitude a and period τ) on the fluid flow, the temperature distribution, the concentration distribution, and the average heat and mass transfers is examined for a given set of the governing parameters, namely the thermal Darcy-Rayleigh number, Ra, the Lewis number, Le, and the ratio of the buoyancy forces, N.

2. PROBLEM FORMULATION

A schematic of the physical problem and the boundary conditions are shown in Figures 1a–1b. It consists of a square saturated porous cavity with the right and left vertical walls thermally insulated and submitted respectively to constant but different concentrations S'_1 and S'_0 ($S'_1 > S'_0$). The hot temperature T'_H of the bottom wall varies sinusoidally with time while that of the cold top wall T'_C ($T'_H > T'_C$) is maintained constant. The top and bottom walls of the enclosure are impermeable to the transport of solute. It is assumed that the third dimension of the enclosure is large enough so that the fluid flow, heat and mass transports can be considered two-dimensional. The porous matrix is considered homogenous and isotropic, and the appropriate Darcy model is used. Soret and Dufour effects are neglected and the binary mixture saturating the porous matrix is modeled as a Boussinesq fluid.

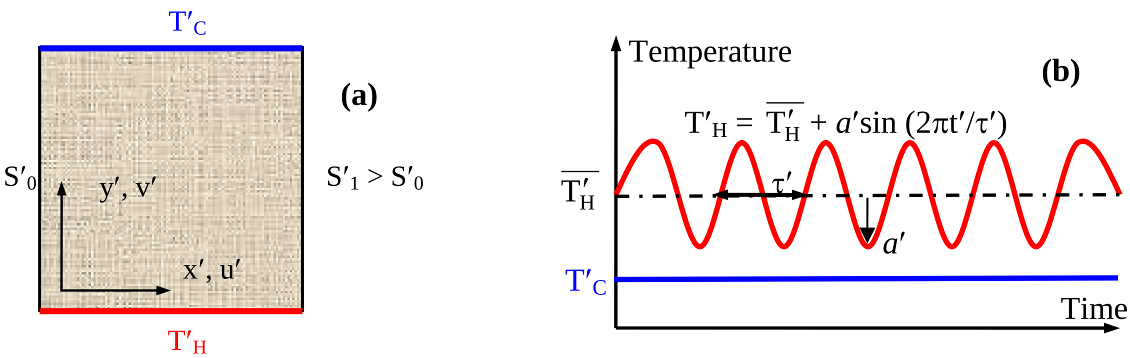


Figure 1: (a) Geometry of the problem and (b) Imposed thermal excitations.

The dimensionless equations governing the problem are:

$$\nabla^2 T = \frac{\partial T}{\partial t} + \frac{\partial (uT)}{\partial x} + \frac{\partial (vT)}{\partial y} \tag{1}$$

$$\frac{\nabla^2 S}{Le} = \varepsilon \frac{\partial S}{\partial t} + \frac{\partial (uS)}{\partial x} + \frac{\partial (vS)}{\partial y} \tag{2}$$

$$\nabla^2 \Psi = -Ra\left[\frac{\partial T}{\partial x} + N\frac{\partial S}{\partial x}\right] \tag{3}$$

$$u = \frac{\partial \Psi}{\partial y} \quad ; \quad v = -\frac{\partial \Psi}{\partial x} \tag{4}$$

In the above equations, the parameters Le, Ra and N denote the Lewis number, the thermal Darcy-Rayleigh number and the solutal to thermal buoyancy ratio, respectively.

The hydrodynamic boundary conditions are such that the stream function is zero on the rigid walls of the cavity ($\Psi = 0$). The dimensionless thermal and solutal boundary conditions associated to the governing equations are:

Left vertical wall: $\frac{\partial T}{\partial x} = 0\,,\; S = 0\,;$ Right vertical wall : $\frac{\partial T}{\partial x} = 0\,,\; S = 1$

Upper horizontal wall: $T = 0\,,\; \frac{\partial S}{\partial y} = 0\,;$ Lower horizontal wall: $T = 1 + a\,sin(2\pi\, t/\tau),\; \frac{\partial S}{\partial y} = 0$

where a and τ are respectively the amplitude and the period of the heating temperature.

The mean Nusselt and Sherwood numbers, characterizing the heat and mass transfers through the heated and salted walls, are averaged in time over periods and evaluated as:

$$Nu = \frac{1}{\tau_{Nu}} \int_0^{\tau_{Nu}} \left(\int_0^1 - \frac{\partial T}{\partial y}\bigg|_{y=0} dx \right) dt \quad ; \qquad Sh = \frac{1}{\tau_{Sh}} \int_0^{\tau_{Sh}} \left(\int_0^1 \frac{\partial S}{\partial x}\bigg|_{x=1} dy \right) dt \qquad (5)$$

where τ_{Nu} and τ_{Sh} represent the periods of the temporal variations of Nusselt and Sherwood numbers respectively.

3. NUMERICAL METHOD

The conservation equations are approximated by finite difference equations with a second order finite difference formula. An alternating direction implicit (ADI) procedure was used to perform the time integration of the energy and mass equations (Eqs. 1 and 2). The stream function equation (Eq. 3) was solved by a point successive over relaxation method (PSOR) with an optimum over-relaxation coefficient equal to 1.939 for the uniform grid of 101×101 adopted in the present study. A convergence criterion was adopted for the stream function to satisfy a variation by less than 10^{-5} at each time step. The velocities at all grid points were determined with Eq. (4) using updated values of the stream function. To check the effect of the grid size, preliminary tests were conducted for different combinations of the governing parameters. The grid of 101×101 was estimated to be appropriate for the present study since it allows a good compromise between the computational cost (a significant reduction of the execution time) and the accuracy of the obtained results. In fact, the results induced by this grid differ by less than 1 % (in the worst case) from those obtained with a refined grid of 151×151. Finally, the accuracy of the numerical model was checked by comparing results from the present investigation with those previously published by Goyeau et al. [11] in the case of double diffusive natural convection in a square porous enclosure submitted to temperature and concentration gradients. The computed heat and mass transfer rates were found to be in excellent agreement with those of the cited reference with maximum relative differences being less than 2%.

4. RESULTS AND DISCUSSION

The results obtained for different values of the amplitude of the variable temperature ($0 \le a \le 1$) and its period ($0.01 \le \tau \le 1$) are presented in this section for a thermal Darcy-Rayleigh number maintained at 100 and a Lewis number equal to 10. The ratio of the buoyancy forces N was fixed at 0.1 (value below the critical threshold of N above which the multiplicity of solutions is not observed for the present problem). More derails concerning the choice of this value of N are given in the study by Bourich et al. [5] who considered the case of double diffusive natural convection in a porous cavity subjected to vertical and horizontal constant gradients of temperature and concentration, respectively. In the reference [5], the multiplicity of solutions was among the objectives of the study while here, attention is mainly focused on the monocellular flow which is maintained for the given values of the parameters Ra, Le and N.

4.1. Streamlines, isotherms and isoconcentration

Typical results of streamlines, isotherms and isoconcentrations, illustrating the monocellular flow and obtained in the case of a constant heating condition ($T_H = 1$) are presented in Figure 2 for Ra = 100, N = 0.1 and Le = 10. It can be seen that the resulting flow patterns consist of a monocellular counterclockwise (trigonometric) cell occupying the entire cavity. The corresponding isotherms show important temperature gradients in the vertical direction near the horizontal walls. This explains the importance of heat transfer from the heated wall of the cavity toward the cold upper one. It should be noted that the decrease of Le (results not presented) does not engender significant changes in the flow structure and temperature distribution but it reduces the development of concentration boundary layers at the level of the vertical walls. In the case of variable heating temperature, the numerical calculations were conducted using the monocellular solutions, obtained for constant heating temperature, as initial condition in order to rule out the other types of solutions (bicellular and possibly tricellular). The resulting flow varies periodically in time with a period identical to that of the thermal excitation, while the amplitudes of the oscillations undergo significant changes when the governing parameters are varied. However, the flow structure remains monocellular throughout the evolution of the cycle, with only a periodic variation of the intensity of the convective cell.

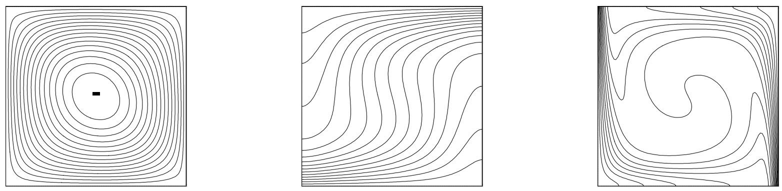
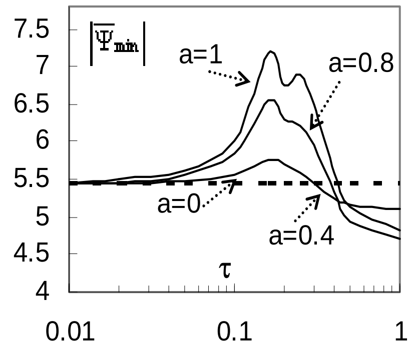


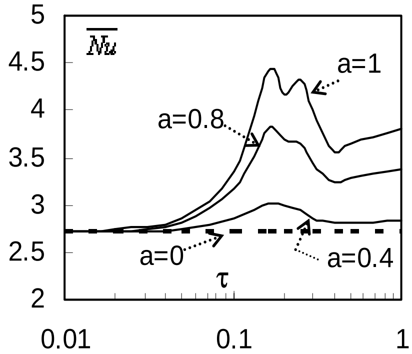
Figure 2: Streamlines, isotherms and isoconcentrations obtained for Ra = 100, N = 0.1 and Le = 10.

4.2. Effect of the period and the amplitude on the time averaged values

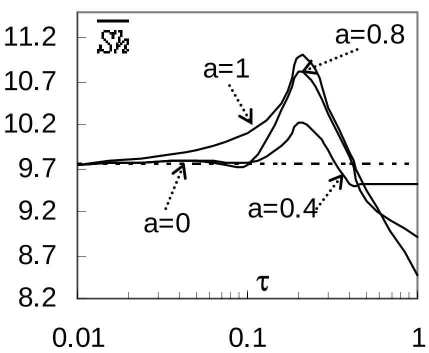
The effect of the period of the exciting temperature requires the variation of this parameter (the period) in a wide range by taking small steps in order to identify correctly an eventual resonance phenomenon within the porous medium. Hence, variations of the time averaged values of the minimum stream function, the Nusselt and Sherwood numbers ($\left|\overline{\Psi}_{min}\right|$, $\overline{Nu}$ and $\overline{Sh}$) are presented respectively in figures 3a–3c for Ra = 100, N = 0.1, Le = 10 and different values of the amplitude (*a* = 0.4, 0.8 and 1). The values corresponding to the case of the isothermal heating source (*a* = 0) are also presented in the same figures as references. It is to be specified that imposed thermal excitations with periods $\tau \leq 0.03$ have no significant effect on these functions since their values remain practically identical to the ones obtained in the regime without thermal modulation. Thereafter, the increase of the flow intensity $\left|\overline{\Psi}_{min}\right|$ with τ is accompanied by an improvement of heat transfer ($\overline{Nu}$) until reaching maximum values for a critical value of $\tau \approx 0.16$. It should be noted that the increase of the oscillating amplitude *a* of the variable temperature does not affect this critical value of τ, but it enhances the amplitude of the peaks (the increase of the peak is about 32% for *a* = 1) in comparison with the case of the steady regime. The existence of a second peak of less importance can be noted for *a* = 1 and τ = 0.25. A further increase of the period leads to a decrease in the variations of $\left|\overline{\Psi}_{min}\right|$ and $\overline{Nu}$ toward values which vary slightly (a slight decrease in the case of $\left|\overline{\Psi}_{min}\right|$ and a slight increase in the case of $\overline{Nu}$) with the period of the exciting temperature when the latter exceeds 0.5. Variations of $\overline{\mathrm{Sh}}$, presented in figure 3c, show that the resonance phenomenon is reached for a critical value of τ = 0.2. The enhancement with respect to the steady state regime increases with *a* to reach 12.55% for *a* = 1.



(a)

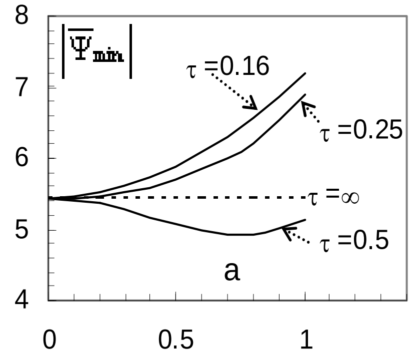


(b)

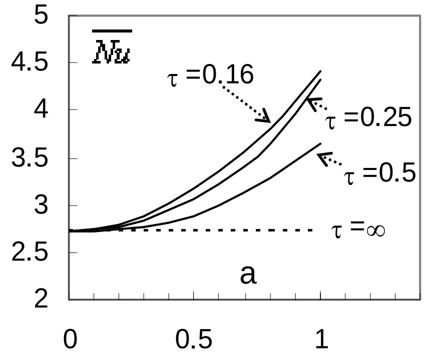


(c)

Figure 3: Effect of the period τ for *Ra* = 100, *N* = 0.1, *Le* = 10 and different *a*: a) $\left|\overline{\Psi}_{\mathrm{min}}\right|$, b) $\overline{Nu}$ and c) $\overline{\mathrm{Sh}}$.



(a)



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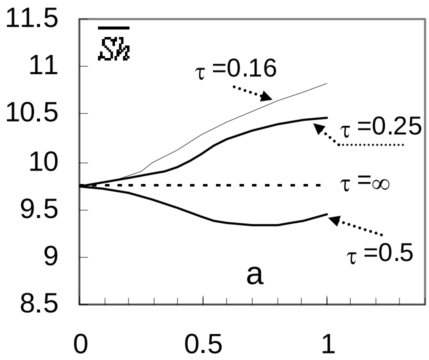

(c)

Figure 4: Effect of the amplitude *a* for *Ra* = 100, *N* = 0.1, *Le* = 10 and different τ: a) $\left|\overline{\Psi}_{\mathrm{min}}\right|$, b) $\overline{\mathrm{Nu}}$ and c) $\overline{\mathrm{Sh}}$.

The effect of the amplitude on $\left|\overline{\Psi}_{\min}\right|$, $\overline{\mathrm{Nu}}$ and $\overline{\mathrm{Sh}}$ is presented respectively in figures 4a–4c for Ra = 100, N = 0.1, Le = 10 and different values of the period τ (τ = 0.16, 0.25 and 0.5). These periods were selected in a range which leads to significant variations of these functions with respect to the steady state case. All the presented functions increase with the increase of the amplitude *a* if we except the variations of $\left|\overline{\Psi}_{\min}\right|$ and $\overline{\mathrm{Sh}}$ for τ = 0.5 (values of τ above this threshold were already identified as being not beneficial).

5. CONCLUSION

A numerical study of double diffusive natural convection has been carried out in the case of a porous square cavity subjected respectively to vertical and horizontal gradients of temperature and concentration and heated with a time periodic temperature. The results obtained show that this mode of heating leads to significant changes in terms of heat and mass transfers within the cavity. The variation of the period of the imposed temperature can improve the heat transfer and the flow intensity in comparison with the case corresponding to a constant heating. The resonance phenomenon is obtained in the present study for a critical period of the exciting temperature that is found to be independent of the Lewis number. This phenomenon is intensified by increasing the amplitude of the exciting temperature.

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