NUMERICAL STUDY OF NATURAL CONVECTION IN A HORIZONTAL CHANNEL PROVIDED WITH HEAT GENERATING BLOCKS

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ABSTRACT

Flow and heat transfer induced by natural convection in a horizontal channel provided with heating blocks periodically mounted on its lower adiabatic surface are studied numerically. The parameters of the study are the ratio of solid blocks to fluid thermal conductivities ($0.1 \le k^* = k_s/k_a \le 200$), and the Rayleigh number ($10^4 \le Ra \le 10^7$). Two models are considered in this study depending whether the blocks are generating uniform heat (model 1) or maintained isothermal at the average temperature calculated using the first model (model 2). The effect of the controlling parameters on the validity of the model with isothermal blocks is examined.

NOMENCLATURE

c^*	Ratio of solid to fluid thermal capacities, $(\rho C_p)_s/(\rho C_p)_a$
k^*	Ratio of solid block to fluid thermal conductivities, (k_s/k_a)
Q	Dimensionless total heat leaving the block surface.
Ra	Rayleigh number, $(g\beta H'^5 q')/(k_a \nu \alpha_a)$
$T_{a(s)}$	Dimensionless temperature, $(T_{a(s)}' - T_F')/(qH'^2/k_a)$
ψ	Dimensionless stream function, (ψ/α).

1. INTRODUCTION

The problem of thermal convection, induced in a channel provided with heating blocks, has received considerable attention due to its implication in the cooling of electronic components (miniature architecture of the channel), in the systems of producing nuclear and chemical energy, in energy storage systems, in solar energy collection and many others. Generally, in this kind of system, there is a volumetric heat generation in the blocks (case of volumetric heat dissipation in the electronic devices or that engendered by chemical or nuclear reactions). However, many previous works considered the blocks as isothermal sources [1-5] which avoid the resolution of energy equation in the blocks and hence reduce computing time and storage space. The literature review shows that most of the works published on thermal convection in a channel provided with blocks are concerned with forced and mixed convection. Nevertheless, natural convection is receiving an increasing attention as it may be efficient in the cooling of electronic systems with weak heat generation and may have undesirable effects in energy storage systems. Conscious of these advantages, several works have been dedicated to the study of natural convection in vertical [5-7] and horizontal [8-9] channels.

Our contribution in the present paper consists in studying natural convection in a horizontal channel provided with volumetric generating heating blocks in order to assess the validity of the isothermal blocks model for this kind of system when the ratio of solid block to fluid thermal conductivities is varied.

2. CONFIGURATION AND MATHEMATICAL MODEL

The system considered in this study is sketched in Fig. 1. It consists of a horizontal infinite channel of height H', with the upper surface maintained at a constant cold temperature. Heating blocks (of height h') are periodically mounted on its lower adiabatic surface. Due to the periodic nature of the system and boundary conditions imposed to the channel, the numerical calculations were restricted to the representative module limited by the fictive planes P_1 and P_2. Two models are examined in this study: in the first one the blocks are submitted to a uniform volumetric heat generation, in the second, the blocks are maintained isothermal (their temperature is an average temperature calculated using the model 1).

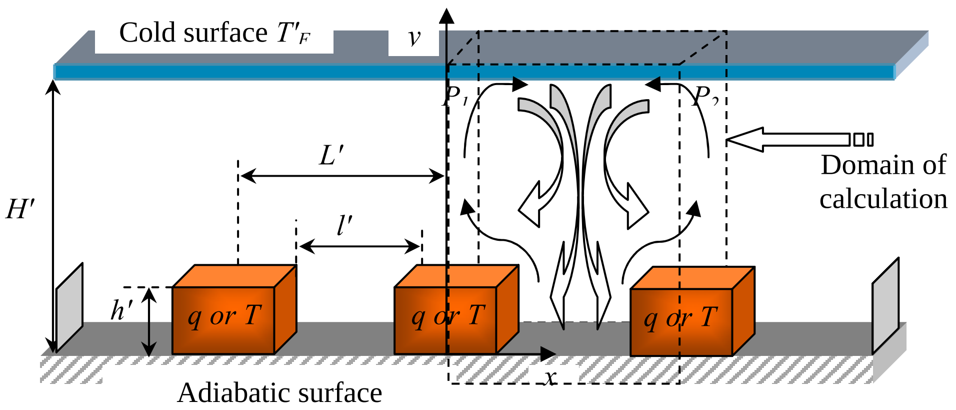


Figure 1: *Studied configuration*

The dimensionless equations governing the problem are written in the stream function-vorticity formulation. The fluid (air) is assumed incompressible and obeying the Boussinesq approximation. The energy equation respectively in the fluid flow and the solid are given by:

$$\frac{\partial T_a}{\partial t} + \frac{\partial u T_a}{\partial x} + \frac{\partial v T_a}{\partial y} = \frac{\partial^2 T_a}{\partial x^2} + \frac{\partial^2 T_a}{\partial y^2} \qquad (1)$$

$$\left(\frac{c^*}{k^*}\right)\frac{\partial T_s}{\partial t} = \frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial y^2} + \frac{1}{k^*} \qquad (2)$$

The conditions of periodicity are applied to the system on the fictive planes P_1 and P_2. For blocks-air interfaces, the continuity of both temperature and heat flux is used for model 1. For the second model (model 2), the blocks are maintained at the average volumetric temperature deduced from the model 1.

The quantity of heat, Q, released by the surface of the block is given by:

$$Q = \int_0^{\frac{1}{4}} \left.\frac{\partial T}{\partial y}\right|_{y=\frac{1}{2}} dx + \int_0^{\frac{1}{2}} \left.\frac{\partial T}{\partial x}\right|_{x=\frac{1}{4}} dy + \int_0^{\frac{1}{2}} \left.\frac{\partial T}{\partial x}\right|_{x=\frac{3}{4}} dy + \int_{\frac{3}{4}}^{1} \left.\frac{\partial T}{\partial y}\right|_{y=\frac{1}{2}} dx \qquad (3)$$

4. RESULTS AND DISCUSSION

The results presented in this study are obtained for Ra ranging from 10^4 to 10^7 and a ratio of thermal conductivities varying in the range $0.1 \leq k^* = k_s/k_a \leq 200$. The choice of the upper and lower limits of k^* is based on the values existing in the literature [9-11]. The geometric parameters and the Prandlt number are considered constant ($A = L'/H' = 1$, $B = h'/H' = 1/2$, $C = l'/H' = 1/2$ and $Pr = 0.72$).

4.1 Flow and temperature fields

4.1.1 Case of Ra = 10^4

For this relatively low value of Ra, the flow regime is globally dominated by conduction. The solution resulting from model 2 is characterized by a symmetrical bicellular flow for the entire range of k^*. However, for model 1, two different structures are obtained (depending on the values of k^*). More precisely, for model 1, symmetrical quadracellular patterns are obtained for relatively small values of k^* ($0.1 \leq k^* < 0.5$). This flow structure disappears in favor of symmetrical bicellular patterns in the range $0.5 \leq k^* \leq 200$. The various structures observed are illustrated in Fig. 3a-e where it is noted that the flow intensity is very weak (weak values of ψ_{ext}) and the isotherms in the fluid phase are horizontal in the upper part of the channel for both models; such a behavior characterizes the pseudo-conductive regime for the present system. For model 1, the tightening of isotherms in the blocks, observed in Fig. 3a for $k^* = 0.1$, results from their low thermal conductivity. Furthermore, the bottom part of the blocks is warmer than their upper part; which is probably the origin of the existence of the quadracellular flow for this case. This presumption is supported by the observations noted while increasing k^*. In fact, by progressively increasing k^*, the spacing between the isotherms increases inside the blocks and the bicellular structure is progressively reconstituted (Fig. 3b and 3c).

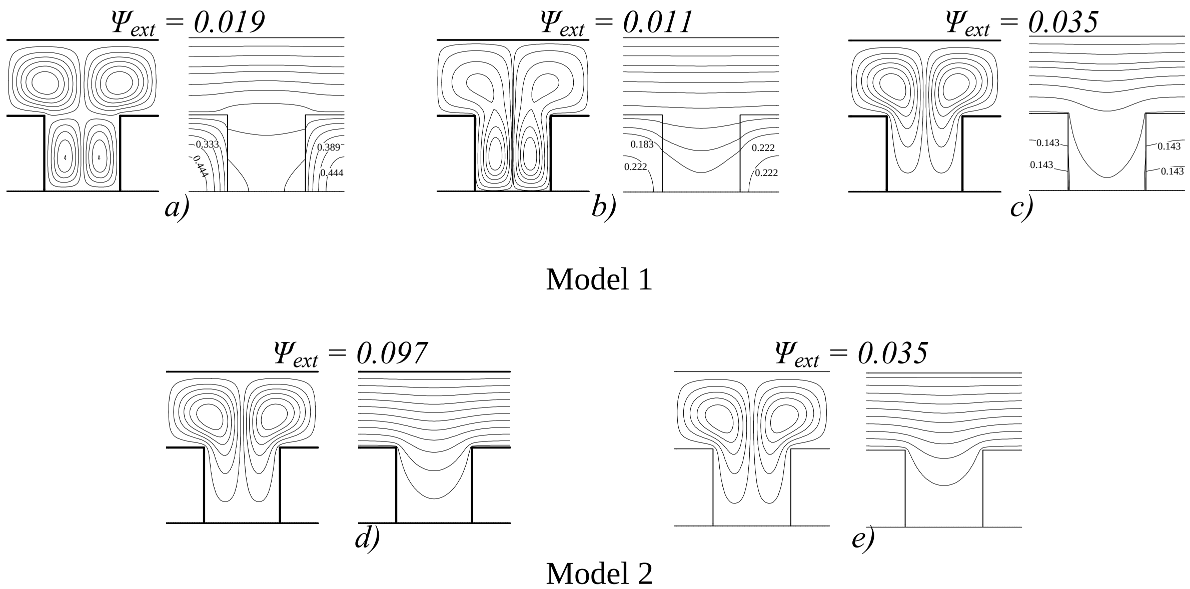
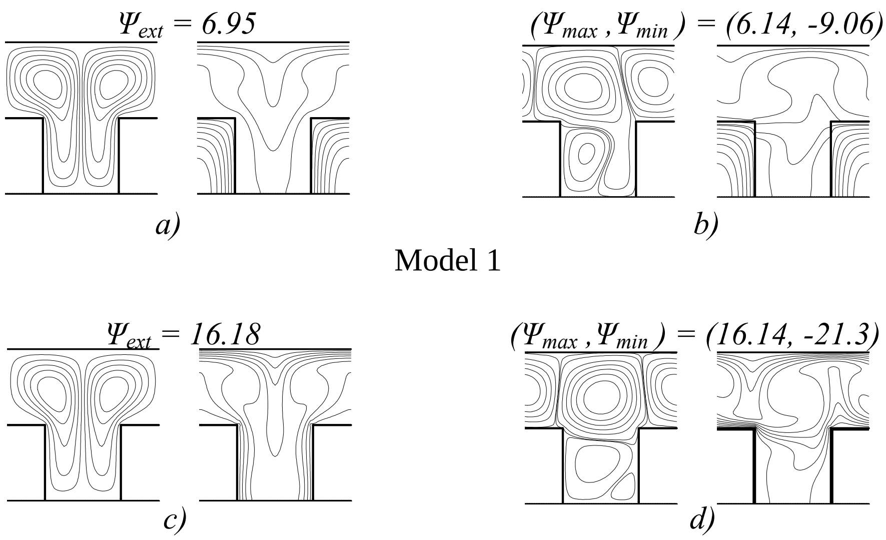


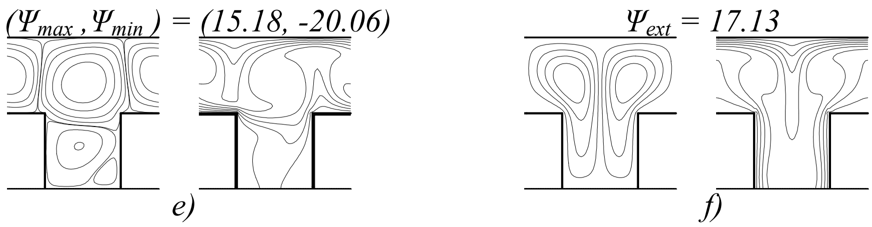
Figure 3: *Streamlines and isotherms obtained for Ra = 10^4 and different values of k*: a) k*= 0.1, b) k*= 0.5, c) k*= 200, d) k*= 0.1 and e) k*= 200.*

4.1.2 Case of Ra = 10^6

At $Ra = 10^6$, the convective effects are dominant and favor the multiplicity of solutions. In fact, a symmetrical bicellular solution (denoted $SS1$) and a dissymmetrical one (denoted $DS1$) are obtained for the model 1. These solutions, illustrated in Fig. 4, lead to different average temperatures for the block which in turn, leads to two different solutions for the model 2. Then four different solutions are obtained in the case of model 2 for a set of governing parameters. To distinguish between these solutions, the symmetrical and dissymmetrical ones of the model 2 based on the average temperatures calculated from symmetrical (dissymmetrical) solutions of model 1 are denoted by $SS2(S)$ and $DS2(S)$ ($SS2(D)$ and $DS2(D)$), respectively. All the symmetrical solutions obtained in this case are bicellular, while the dissymmetrical solutions are multicellular. The quadracellular structure observed for $Ra = 10^4$ disappears for $Ra = 10^6$. For the model 1, the structure of the bicellular flow does not undergo significant qualitative transformations by increasing k^* and the flow intensity varies slightly with this parameter. In fact, as the quantity of heat leaving the blocks remains constant by varying k^*, the temperature at the surface of the blocks increases in a region and decreases in the other in such away that the total quantity of heat evacuated remains unchanged. This behavior of the temperature around the block explains the weak variation of the flow intensity with k^* in the case of the model 1. As indication, ψ_{ext} varies from 6.96 to 7.27 (a difference of about 4.4 %) when k^* is increased from 0.1 to 200 for a symmetrical solution. For the model 2, the effect of k^* on ψ_{ext} is considerable; its value drops from 16.18 for $k^* = 0.1$ to 7.28 for $k^* = 200$. The streamlines and isotherms of Fig. 4a-f illustrate different solutions obtained for $k^* = 0.1$. Although the flows structures are qualitatively similar for both models, the convective cells generated by the model 2 are much more intense than those corresponding to the model 1. For $k^* = 200$ (results not presented here), the flow structure remains unchanged (in comparison with the case of $k^* = 0.1$). However, the recirculating cells undergo a slight displacement towards the right side in the case of the dissymmetrical solutions corresponding to the model 1 with a small change in the size for the cells in the micro cavity. Quantitatively, for $k^* = 200$, the intensities of the symmetrical cells generated by both models are nearly the same (the difference is less than 4%). The same deduction can be made in the case of the dissymmetrical solutions.

It should be noted that the multiplicity of solutions persists in the case of $Ra = 10^7$. The symmetrical bicellular structures become slightly dissymmetrical at relatively weak values of k^* (results not presented). For the model 2, the symmetrical solution $SS2(D)$ / $SS2(S)$ is not obtained in the range $0.1 \leq k^* < 1$/($0.1 \leq k^* < 0.43$).





Model 2

Figure. 4: Streamlines and isotherms obtained for Ra = 10^6 and k= 0.1*
a) SSM_1, b) SDM_1, c) $SSM_2(S)$, d) $SDM_2(D)$, e) $SDM_2(S)$ and f) $SSM_2(D)$

4.2 Heat transfer

To compare the quantity of heat leaving the blocks for both models, we present in Fig. 5a-b the variations of Q versus k^* for $Ra = 10^4$ and 10^6 and various solutions. It can be noted that the heat transfer is always overestimated by the model 2 (isothermal model) for weak values of k^*. The limit value of k^* (denoted by k_L^*) above which this model reproduces the heat transfer results corresponding to the model 1 is calculated for different Ra and different solutions. The criteria adopted in the comparison is such that the difference between the quantities of heat generated by both models remains within 5%. The results obtained show that, for $Ra = 10^4$, the value of k_L^* is about 5.9 but this threshold of k_L^*, when it exists, changes from one type of solution to another. More precisely, for $Ra = 10^6$ (10^7) k_L^* is about 14.7 (15) and 19 (35), respectively for the solutions $SS2(S)$ and $DS2(D)$. However, the differences of the quantities of heat generated by the solutions $SS2(D)$ and $DS2(S)$ of the model 2, compared to those of the first model, remain always higher than 5 %; a nearly constant difference persists between these quantities even at very large values of k^*. These differences are less than 6% in the case of $Ra = 10^6$, but reach 13% for $DS2(S)$ solutions and 14% for $SS2(D)$ solutions for $Ra = 10^7$. This means that the disagreement between the two models is accentuated with the increase of Ra.

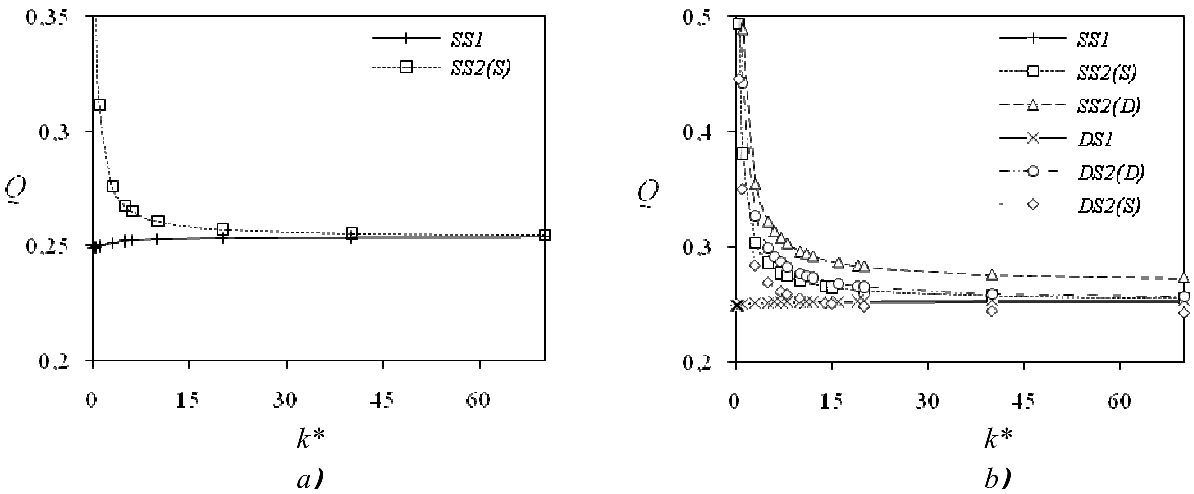


Figure 5*: Variations of Q with k * for different Ra: a) Ra= 10^4 and b) Ra= 10^6.*

5. CONCLUSION

Comparison between the model of blocks with uniform heat generation and the model of isothermal blocks leads to the following conclusions:

- The conductivity ratio k^* has a considerable effect on the results of both models.
- The heat flux as well as the intensity of the flow are found to be overestimated by the model 2 (isothermal model).
- In the case of $Ra = 10^4$, the model 1 generates quadracellular flow structures for weak values of k^*, while the model 2 generates only bicellular flow structures.
- A multiplicity of solutions was obtained in the case of the first model for $Ra = 10^6$ and 10^7 which leads to four solutions in the case of model 2.
- In the case of the multiplicity of solutions and sufficiently large values of k^*, the model 2 allows the reproduction (in terms of heat transfer) of the results obtained with model 1 for some solutions but for the others, a constant difference is maintained (the difference may reach 14%).

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