COUPLING BETWEEN NATURAL CONVECTION AND RADIATION IN A SQUARE CAVITY SUBMITTED TO CROSS GRADIENTS OF TEMPERATURE

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ABSTRACT
The present paper reports numerical results of natural convection and surface radiation within a horizontal square cavity filled with air and discretely heated and cooled from the four walls. The horizontal and vertical opposite walls are differentially heated. In this study, the bottom active wall is at higher temperature than that of the top active wall. The remaining portions of the walls are considered adiabatic. The parameters governing the problem are the emissivity of the walls ($0 \leq \varepsilon \leq 1$) and the relative dimensions of the active elements ($0.1 \leq B = h'/H' = \ell'/L' \leq 0.9$). Results of the study show that radiation has no effect on the convective heat transfer across the cavity but leads to enhance significantly the total Nusselt number with respect to the case of pure natural convection.

1. INTRODUCTION

Fluid flow and heat transfer induced by natural convection in rectangular closed cavities have been the object of interest in the past owing to their practical importance in many engineering applications. Actually, considerable efforts are still devoted in the area where this basic geometry is a center of attraction. The interest in such problems stems from their importance in many engineering applications such as convective heat loss from solar collectors, thermal design of buildings, air conditioning and recently, the cooling of electronic components. In applications involving natural convection as a removal heat transfer mechanism, radiation effects are often neglected. However, thermal radiation is present in general and it is strongly coupled with natural convection; its effect could be neglected only in geometries with polished surfaces. In the literature, most of the existing studies on the coupling between natural convection and surface radiation are concerned with rectangular cavities where the temperature gradient is either horizontal or vertical, including different kinds of boundary conditions [1-6]. The results obtained show that, despite its negative but limited effect on natural convection, the walls' radiation supports the overall heat transfer. Most of the scientific works conducted in the past on natural convection coupled with radiation inside rectangular enclosures has been substantially oriented to study unidirectional heat flows engendered by imposed heat fluxes or temperature differences either in horizontal or vertical directions. Actually, much more complex boundary conditions may be encountered in practical cases where cross gradients of temperature are imposed to the cavity.

To the best knowledge of the authors, there appears to be no studies in the literature concerned with natural convection coupled with radiation in square cavities submitted to cross gradients of temperature. Hence, the objective of this study consists to investigate numerically the coupling between natural convection and radiation in a square cavity partially heated from below and from side while the opposite elements of the heated surfaces are partially cooled. The effect of the emissivity of the walls, ε, and the relative length of the active walls, B, on the dynamical and thermal behaviors is considered. A special attention is given to quantify the contribution of convection and radiation to the overall heat transfer.

2. PROBLEM FORMULATION

The configuration under study is depicted in Fig. 1. It consists of a square cavity partially heated from below and from the left vertical side and submitted to cross gradients of temperature (partial cooling from above and from the right vertical side) with the assumption that the horizontal and vertical temperature differences are identical. All the thermally active surfaces are centrally located. In addition, the inner surfaces of the cavity are assumed to be gray, diffuse emitters and reflectors of radiation with identical emissivities. The flow is conceived to be laminar and incompressible. All the thermophysical properties of the fluid are assumed constant except the density in the buoyancy term which is assumed to vary linearly with temperature. The non-dimensional governing equations, written in Ω-Ψ formulation, are as follows:

$$\frac{\partial \Omega}{\partial t}+\frac{\partial (u\Omega)}{\partial x}+\frac{\partial (v\Omega)}{\partial y}=Ra\,Pr\frac{\partial T}{\partial x}+Pr\left(\frac{\partial^2 \Omega}{\partial x^2}+\frac{\partial^2 \Omega}{\partial y^2}\right) \qquad (1)$$

$$\frac{\partial T}{\partial t}+\frac{\partial (uT)}{\partial x}+\frac{\partial (vT)}{\partial y}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2} \qquad (2)$$

$$\frac{\partial^2 \Psi}{\partial x^2}+\frac{\partial^2 \Psi}{\partial y^2}=-\Omega \qquad (3)$$

$$u=\frac{\partial \Psi}{\partial y}\ ,\ v=-\frac{\partial \Psi}{\partial x} \quad \text{and} \quad \Omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y} \qquad (4)$$

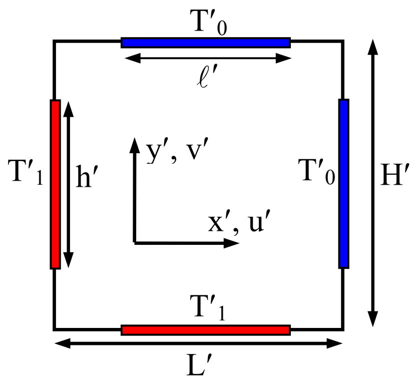


Figure 1: (a) Geometry of the problem with the imposed thermal excitations.

2. 1. Boundary conditions

The boundary conditions, associated to the problem are as follows:

$u = v = \Psi = 0$ on the rigid boundaries
$T = 0$ on the cooled portions
$T = 1$ on the heated portions
$-\frac{\partial T}{\partial n}+N_r Q_r=0$ on the adiabatic walls

where ″n″ being the normal direction to the considered adiabatic wall.

2. 2. Surface radiation calculations

The calculation of the radiative heat exchange between the internal walls of the cavity is based on the radiosity method. The grid used for convection (81×81) is retained in the presence of radiation and consists of 324 isothermal elementary surfaces. Each segment is sufficiently short to be considered isothermal. The view factors between the elementary surfaces were determined by the Hottel's [7] crossed string method. The inner surfaces of the enclosure are assumed to be opaque, diffuse and gray. The non-dimensional radiosity and the net radiative heat flux for the ith element of the enclosure are evaluated respectively by:

$$J_i = \varepsilon_i \left(\frac{T_i}{T_o} + 1 \right)^4 + (1-\varepsilon_i) \sum_{S_j} F_{ij} J_j \quad ; \qquad Q_r = J_i - I_i = \varepsilon_i \left[\left(\frac{T_i}{T_o} + 1 \right)^4 - \sum_{S_j} F_{ij} J_j \right] \tag{5}$$

2. 3. Heat transfer

The average convective and radiative Nusselt numbers, evaluated for the heated walls, are defined as:

- On the vertical heated wall:

$$Nu_V(cv) = -\int_{0.5-\frac{B}{2}}^{0.5+\frac{B}{2}} \left(\frac{\partial T}{\partial x} \right)\bigg|_{x=0} dy \quad ; \quad Nu_V(rd) = \int_{0.5-\frac{B}{2}}^{0.5+\frac{B}{2}} \left(N_r Q_r \right)\big|_{x=0} dy \tag{6}$$

- On the horizontal heated wall:

$$Nu_H(cv) = -\int_{0.5-\frac{B}{2}}^{0.5+\frac{B}{2}} \left(\frac{\partial T}{\partial y} \right)\bigg|_{y=0} dx \quad ; \quad Nu_H(rd) = \int_{0.5-\frac{B}{2}}^{0.5+\frac{B}{2}} \left(N_r Q_r \right)\big|_{y=0} dx \tag{7}$$

The average convective and radiative Nusselt numbers across the hole cavity are defined respectively as $Nu(cv) = Nu_V(cv) + Nu_H(cv)$ and $Nu(rd) = Nu_V(rd) + Nu_H(rd)$. In addition, the average Nusselt number *Nu* of the whole enclosure is calculated as $Nu = Nu(cv) + Nu(rd)$.

3. METHOD OF SOLUTION

The non linear partial differential governing equations, Eqs. (1)-(3), were discretized using a finite difference technique. The first and second derivatives were approached by central differences. The integration of equations (1) and (2) was ensured by the Alternating Direction Implicit method (ADI). At each time step, the Poisson equation, Eq. (3), was treated by using the Point Successive Over-Relaxation method (PSOR). The set of Eqs. (5) was solved by using the Gauss-Seidel method. The numerical code was validated against the results of Akiyama and Chong [1] obtained in the case of a square cavity differentially heated. Comparisons, made in terms of convective Nusselt numbers, evaluated at the heated wall, showed a fairly good agreement with relative maximum deviations limited to 1.07 % / (1.36 %) for $\varepsilon = 0$ / (1) as seen in Table 1.

Table 1. Effect of Ra and ε on the mean convective Nusselt number, Nu_{cv}, evaluated on the heating wall of a square cavity for T'_H = 298.5 K and T'_C = 288.5 K.

	ε = 0				ε = 1			
Ra	**10^3**	**10^4**	**10^5**	**10^6**	**10^3**	**10^4**	**10^5**	**10^6**
Present work	1.118	2.257	4.627	9.475	1.250	2.242	4.192	8.100
Akiyama and Chong [1]	1.125	2.250	4.625	9.375	1.250	2.250	4.250	8.125

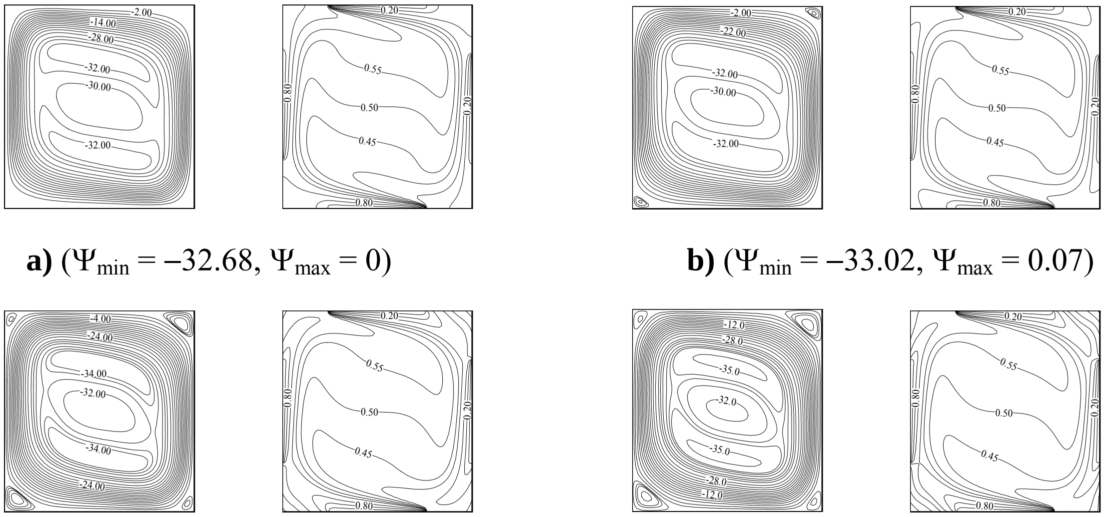
4. RESULTS AND DISCUSSION

In the following, effects of relative length of the active walls, $0.1 \leq B \leq 0.9$, and walls' emissivity, $0 \leq \varepsilon \leq 1$, on fluid flow and heat transfer characteristics are illustrated. The results presented were obtained with $Ra = 10^6$ and by using air as a working fluid ($Pr = 0.72$).

The effect of radiation on the flow structure and temperature distribution inside the cavity is illustrated in Figs. 2a-2d for B = 0.5 and various values of ε. The analysis of the streamlines presented in Fig. 2a, in the absence of walls' radiation (ε = 0), reveals the existence of a main clockwise unicellular flow in contact with all the thermally active surfaces, surrounding three small cells. The corresponding isotherms are tightened at the level of the horizontal active walls indicating a good convective heat exchange between these walls and the fluid. In the vicinity of the heated and cooled vertical walls, the isotherms are more spaced indicating a less convective heat exchange between these surfaces and the fluid. This behavior is explained by the clockwise rotation of the fluid. In fact, the fluid moving upward/downward along the vertical left/right wall was already heated/cooled by the lower/upper horizontal wall and reduces consequently the heat transfer by convection across the vertical active walls. A quantitative comparison shows that the convective heat transfer along the heated horizontal wall is approximately twice of that along the vertical heated one. When the effect of radiation is considered for highly polished walls (ε = 0.05), Fig. 2b shows that the flow recovers a state characterized by an intensification of the central cells and the appearance of two small cells at the corners localized between the heated and the cooled walls. A further increase of ε to 0.5 and 0.85 (highly emissive walls) leads to more complicated flow structure characterized by the increase of size and intensity of these cells and also by the formation of two other cells at the remaining corners of the cavity (Figs. 2c-2d). It is to mention that the strength of these cells depends on the relative dimension of the active walls; the maximum sizes and intensities are obtained for B = 0.6. Finally, it is to precise that the centro-symmetry observed in the flow structure in pure natural convection, is preserved in the presence of radiation.

Variations, versus the relative length of the active walls B, of the average Nusselt numbers, resulting from contributions of convection and radiation and total Nusselt number, evaluated along the heated walls, are presented in Figs. 3a–3c for various values of ε. Fig. 3a shows a monotonous increase of Nu(cv) with B either with or without radiation effect. This tendency is justified by the flow intensification promoted by the increase of B. It is to be specified that for all the considered values of B, the presented convective Nusselt number is not affected by the increase of the emissivity of the walls, indicating an insensitivity of the convection heat transfer component to radiation effect. This finding is fundamentally different from the well known tendencies characterized by a negative role of radiation on the natural convection heat transfer [1, 2, 4, 5, 6]. A close examination of the variations of the convective Nusselt numbers on the vertical and horizontal heated walls shows that the effect of surface radiation is different on these walls. In fact, the obtained results show that surface radiation leads to an enhancement/drop of the convective component across the vertical/horizontal heated walls. It is important to underline that while modeling the interaction between natural convection and surface radiation in rectangular enclosures submitted to a single temperature gradient (horizontal or vertical), the results previously reported in the literature showed that surface radiation leads to a decrease of the convective Nusselt number. The effect of the emissivity of the walls on the radiative heat transfer component is presented in Fig. 3b in terms of Nu(rd) variations with B for ε = 0.05, 0.5, 0.85 and 1. Globally, it can be deduced that, for a given value of B, the effect of radiation is important for ε ≥ 0.5 and it is characterized by an important increase of Nu(rd) with ε. Also, the effect of B on Nu(rd) is limited in the case of a highly polished aluminum sheet (ε = 0.05) but becomes increasingly positive by increasing ε. More precisely, for ε = 0.5/(1), Nu(rd) increases drastically by about 900%/(1070%)

when the value of B passes from 0.1 to 0.9. The variations of the total Nusselt number with B, presented in Fig. 3c, show increasing tendencies of Nu with B and ε.



a) ($\Psi_{min} = -32.68$, $\Psi_{max} = 0$) **b)** ($\Psi_{min} = -33.02$, $\Psi_{max} = 0.07$)

c) ($\Psi_{min} = -34.66$, $\Psi_{max} = 0.74$) **d)** ($\Psi_{min} = -35.21$, $\Psi_{max} = 1.27$)

Figure 2: Streamlines and isotherms obtained for Ra = 10^6, B = 0.5 and various values of ε: a) ε = 0, b) ε = 0.05, c) ε = 0.5 and d) ε = 0.85.

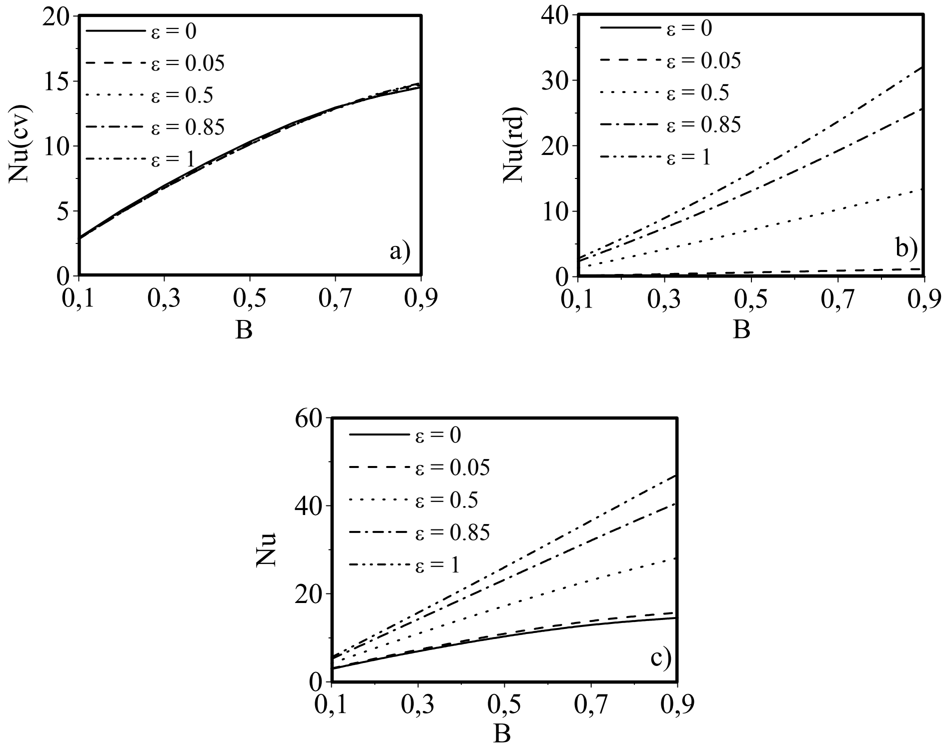


Figure 3: Effect of B on the total Nusselt numbers for Ra = 10^6 and various values of ε : a) Nu(cv), b) Nu(rd) and c) Nu.

The contribution of radiation to the total heat transfer through the heated walls of the cavity is quantified by presenting, in Fig. 4, the evolution of the ratio [Nu(rd)/Nu×100] with B for various values of ε. It is seen from the figure that for ε ≤ 0.05, the contribution of radiation to the total heat transfer remains almost insensible to the increase of B. Above this threshold of ε, the increase of B leads obviously to an increase of the radiation contribution, behavior resulting from the fact that

Nu(rd) increases with B with a rate higher than that of the convective component Nu(cv) (compare Fig. 3a and b). Generally, we can conclude that the heat transferred by radiation is higher than that transferred by convection for $\varepsilon \geq 0.85$. In the case of $\varepsilon \leq 0.5$, the tendency is reversed and convection is found to contribute more than radiation to the overall heat transfer. However, the contribution of radiation is important for moderate and high values of ε. Quantitatively, the minimum contribution of the radiative component, observed at B = 0.1, is about (32%)/(49%) for ε = (0.5)/(1) and increases to about (48%)/(68%) for the higher considered value of B (B = 0.9). This means that the contribution of radiation to the overall heat transfer remains non-negligible.

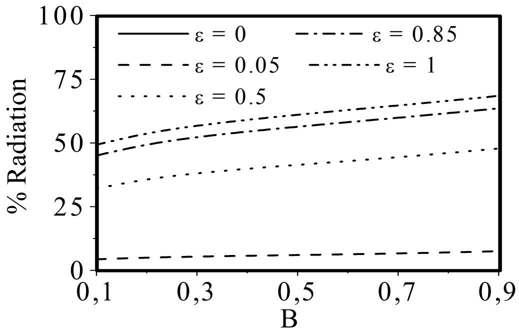


Figure 4: Radiation contribution to the global heat transfer versus B, for Ra = 10^6 and various values of ε.

5. CONCLUSION

Combined natural convection and surface radiation inside a square cavity, submitted to cross gradients of temperature, has been studied numerically. The investigation provides the trends to show the effects of the relative dimension of the active walls B and the emissivity ε on the heat transfer and flow structure. Both cases of pure natural convection and natural convection coupled with radiation were considered. It is found that radiation has no effect on the average convective Nusselt number component and the latter is favored by the increase of the relative dimension of the active walls, B. The parameters B and ε are found to have a positive effect on the radiative and total heat transfers. The contribution of radiation to the total heat transfer is generally not negligible for $\varepsilon \geq 0.5$.

REFERENCES

1. M. Akiyama and Q. P. Chong, 1997, Numerical Analysis of Natural Convection with Surface Radiation in a Square Enclosure. *Num. Heat Transfer A*, vol. 31, pp. 419–433.

2. E. H. Ridouane, M. Hasnaoui, A. Amahmid, and A. Raji, 2004, Interaction between Natural Convection and Radiation in a Square Cavity Heated from Below. *Num. Heat Transfer A*, vol. 45, pp. 289–311.

3. C. Gururaja Rao, A. Venkata Krishna, and P. Naga Srinivas, 2005, Simulation Studies on Multimode Heat Transfer from a Square-Shaped Electronic Device with Multiple Discrete Heat Sources. *Num. Heat Transfer A*, vol. 48, pp. 427–446.

4. A. Bahlaoui, A. Raji, and M. Hasnaoui, 2006, Combined Effect of Radiation and Natural Convection in a Rectangular Enclosure Discreetly Heated from One Side. *Int. J. Num. Methods for Heat & Fluid Flow*, vol. 16, pp. 431–450.

5. C. Y. Han and S. W. Baek, 2000, The Effects of Radiation on Natural Convection in a Rectangular Enclosure Divided by Two Partitions. *Num. Heat Transfer A*, vol. 37, pp. 249–270.

6. C. Balaji and S. P. Venkateshan, 1994, Correlations for Free Convection and Surface Radiation in a Square Cavity. *Int. J. Heat Fluid Flow*, vol. 15, pp. 249–251.

7. H. C. Hottel and A. F. Saroffim, 1967, Radiative Heat Transfer, *McGraw-Hill*, New York.