PERTURBATION METHOD APPLIED ON A REACTION-DIFFUSION MODEL FOR FOREST FIRE PROPAGATION: THERMAL CONDUCTIVITY EFFECT

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ABSTRACT

We present a 1D physical reaction-diffusion model for forest fire propagation involving five free boundaries and including several physical processes such as drying, heat convection, heat conduction, and radiative transfer.

In this paper, we consider only the three first zones, just before the fire front. In these zones, the thermal conductivity of the vegetal phase can be considered as constant. Using dimensional analysis, it is shown that this thermal conductivity can be considered as a small parameter. The rate of spread and the drying zone can be obtained as a perturbation expansion in this parameter.

The rate of spread for a flame model is calculated. We use Padé approximants to locate singularity limiting the range of validity of the series solutions for this model.

NOMENCLATURE

T	vegetation temperature
ρ	mass density
λ	thermal conductivity
R_c	chemical heat source including drying
M	radiation heat source
h	convection coefficient
T_{ext}	external temperature
T_{pyr}	pyrolysis temperature
H_u	humidity
H_{u0}	initial humidity
L_{ev}	evaporation latent heat
Φ	porosity
C_s	heat capacity of the solid constituent of vegetation
C_w	heat capacity of the water.
δ	height of the vegetation
K	extinction coefficient of the vegetation
B	Stefan Boltzman's constant

ε_f	emissivity
T_{fl}	temperature of flame
h_f	height of flame
W	width of flame
T_f	temperature of the gaseous phase
ρ_{ext}	extinction vegetation density
T_i	ignition temperature
$\left[x_{ev}^-, x_{ev}^+\right]$	evaporation zone
T_∞	ambient temperature
u	rate of spread

1. INTRODUCTION

The models of propagation of forest fires are classified in four principal groups: statistical, geometric, empirical and physical models. The last ones are divided into two categories: detailed and reduced models.
The reaction-diffusion model used here is a reduced model and has a generic form:

$$\begin{cases} \rho C \dfrac{\partial T}{\partial t} = \lambda \Delta T + R_c(\rho, T) - h(T - T_{ext}) + M_r \\ \dfrac{\partial \rho}{\partial t} = f(\rho, T) \end{cases} \quad (1)$$

where T is the vegetation temperature, ρ is the mass density, C is the heat capacity and λ is the thermal conductivity. $R_c(\rho, T)$ is the chemical heat source including drying, M_r is a radiation heat source, h is the convection coefficient and T_{ext} is an external temperature. The second equation represents the variation of mass density because of chemical reactions. The parameters of the models have a physical meaning but they should be related to real physical parameters. This kind of modelling assumes that the vegetation is a continuum medium. Furthermore, this system of equations is set on a two-dimensional domain, although the vegetation lies in a three-dimensional domain, and the fire front is recovered as the curve $T(x, y, t) = T_{pyr}$, the temperature T_{pyr} being the pyrolysis temperature.

The radiation flux M_r due to the presence of tall flames is a consequence of the combustion in the region of ambient air. That is, the process is a full three-dimensional process, contrary to the assumption encountered in physical modelling. One of the difficulties is the modelling of what happens at the vegetal stratum level, but several efforts have been done in this direction, cf. [1, 2, 3]. Moreover, a bridge between reaction-diffusion models and detailed models has been obtained, cf. [4].

In this paper we will address the question of the possibility of modelling forest fires propagation by a reaction-diffusion model. Paragraph 2 is devoted to the presentation of such model; as the drying is a complex process, a simplified version of the model is considered. The last paragraph is devoted to the analytical calculation of the rate of spread. It is shown that the obtained rate of spread is closed to the ones obtained experimentally [6].

2. REACTION-DIFFUSION MODEL FOR FOREST FIRE PROPAGATION

It has been demonstrated that a 2D reaction diffusion model could be derived from a detailed combustion model, cf. [4, 9]. The obtained system of equations is of type (1). We may first suppose that evaporation is at a constant temperature T_{ev} and neglect the heat absorbed by solid during pyrolysis, so that the energy equation (1) reduces to:

$$(1-\Phi)\rho(C_s + H_u C_w)\frac{\partial T}{\partial t} = \lambda\Delta T + h(T_{ext} - T) + \rho\frac{\partial H_u}{\partial t}L_{ev}\delta_{T=T_{ev}} + M_r \tag{2}$$

In this relation, the symbol $\delta_{T=T_{ev}}$ stands for the Dirac distribution of the zone $T = T_{ev}$ and H_u is the humidity or moisture content. L_{ev} is an evaporation latent heat, Φ is the porosity, C_s is the heat capacity of the solid constituent of vegetation and C_w is the heat capacity of the water.
For the radiation heat source M_r, we choose the model presented by De Mestre *et* al [10]:

$$M_r = \delta K \varepsilon_f \, BT_{fl}^4 \frac{2}{\pi}\arctan\left(\frac{Wh_f}{X\sqrt{W^2 + h_f^2 + X^2}}\right) \tag{3}$$

where δ is the height of the vegetation, K its extinction coefficient and B is Stefan Boltzman's constant. ε_f and T_{fl} are respectively the emissivity and the temperature of flame, h_f its height, and W its width. X is the distance between a point of the vegetation and the flame.
Now the model can be summed up as follows:

i) In the zone before the evaporation front, denoted by zone I, such that: $T < T_{ev}$ and $\rho > \rho_{ext}$,

$$(1-\Phi)\rho(C_s + H_{u0}C_w)\frac{\partial T}{\partial t} = \nabla\cdot(\lambda\nabla T) + M_r - h(T - T_f) \tag{4}$$

where H_{u0} is the initial humidity, T_f the temperature of the gaseous phase and ρ_{ext} the extinction vegetation density.
ii) In the evaporation zone, denoted by zone II, such that: $T = T_{ev}$, $H_u > 0$ and $\rho \geq \rho_{ext}$,

$$-\rho L_{ev}\frac{\partial H_u}{\partial t} = M_r - h(T_{ev} - T_f) \tag{5}$$

iii) In the intermediary zone between the evaporation zone and the burning zone, denoted by zone III, such that: $T_{ev} < T < T_i$, $H_u = 0$ and $\rho \geq \rho_{ext}$,

$$(1-\Phi)\rho C_s\frac{\partial T}{\partial t} = \nabla\cdot(\lambda\nabla T) + M_r - h(T - T_f) \tag{6}$$

T_i is the ignition temperature.
iv) In the burning zone, denoted by zone IV, such that: $T \geq T_i$, $H_u = 0$ and $\rho \geq \rho_{ext}$,

$$(1-\Phi)\rho C_s\frac{\partial T}{\partial t} = \nabla\cdot(\lambda\nabla T) + M_r - h(T - T_f) \tag{7}$$

The variation of mass due to chemical reactions is:

$$\frac{\partial\rho}{\partial t}(\mathbf{P},t) = -v_r(\mathbf{P},t-t_i)\rho(\mathbf{P},t) \tag{8}$$

t_i is the instant of ignition, and v_r characterizes the speed of the chemical reaction. One can consider an Arrhenius law:

$$v_r(\mathbf{P},t-t_i) = A\exp(-E/RT) \tag{9}$$

where E is the activation energy, A the prefactor and R the gas constant.
v) In the burnt zone, denoted by zone V, such that: $\rho = \rho_{ext}$

$$(1-\Phi)\rho_{ext}C_s \frac{\partial T}{\partial t} = \nabla \cdot (\lambda \nabla T) + M_r - h(T - T_f) \tag{10}$$

All the preceding described regions are illustrated in figure 1.

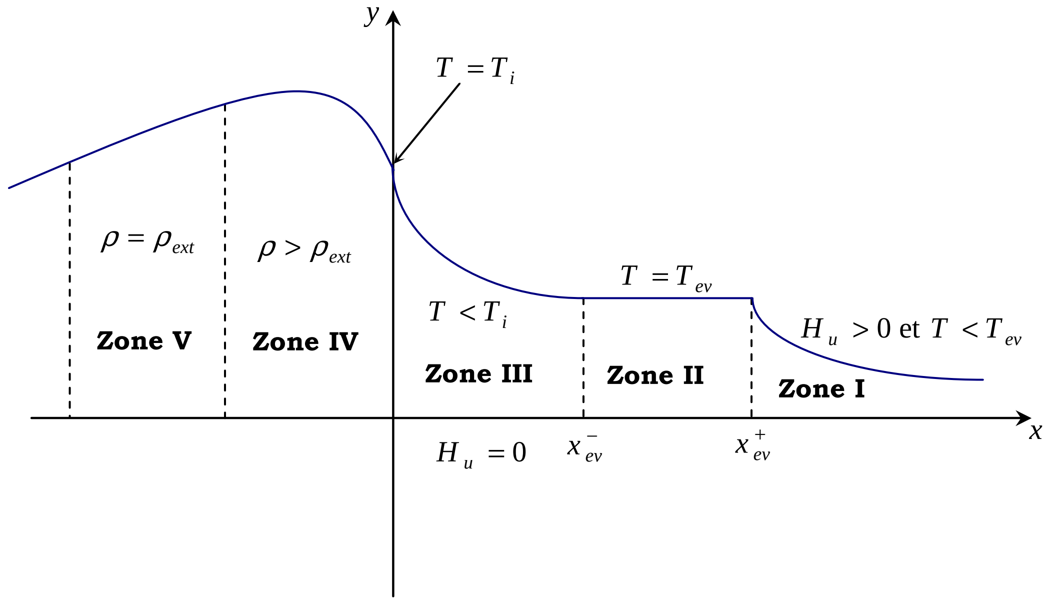

Figure 1. Different zones related to the spreading in one dimensional propagation; the evaporation zone is the interval $\left]x_{ev}^-, x_{ev}^+\right[$.

3. DETERMINATION OF THE RATE OF SPREAD

An interesting feature of the preceding simplified model is the possibility of deriving a system of equations for the rate of spread. Let us consider the three first zones, just before the fire front. In these zones, the thermal conductivity can be considered as constant. Let us introduce the following non-dimensional quantities, where T_∞ is the ambient temperature:

$$\overline{T} = \frac{T - T_\infty}{T_i - T_\infty} \quad , \quad \overline{T}_{ext} = \frac{T_{ext} - T_\infty}{T_i - T_\infty} \tag{11.a}$$

$$M_r = \overline{M}_r M_0 \quad , \quad t = \frac{A(T_i - T_\infty)}{M_0}\bar{t} = \tau\bar{t} \tag{11.b}$$

In relations (11.b), coefficient A is given by $A = (1-\Phi)\rho(C_s + H_{u0}C_w)$ and there is a characteristic time τ defined by $\tau = \frac{A(T_i - T_\infty)}{M_0}$. We also consider a characteristic length l. With these new variables, we obtain the following reduced thermal conductivity $\overline{\lambda} = \frac{\lambda(T_i - T_\infty)}{l^2 M_0}$. For the physical values involved M_0 = 5.2 kW/m², λ = 0.27 W.$m^{-1}.K^{-1}$, T_i = 300 °C, and T_∞ = 27 °C. We thus obtain

the following value for the reduced thermal conductivity $\overline{\lambda} = \dfrac{0.0142}{l^2}$. Therefore, even if $l = O(1)$, $\overline{\lambda}$ is a small parameter.
Let us consider a one-dimensional propagation with a constant non-dimensional velocity *u* (in fact $\overline{u}$ with $u = \dfrac{l}{\tau}\overline{u}$, where *u* is the dimensional velocity). And let us look for a stationary wave solution of the form $T(x-ut)$, $H_u(x-ut)$; we obtain a non linear eigenvalue problem. The following boundary conditions and continuity conditions must be added:

$$T(0)=T_i=1\ ,\ T(\infty)=0 \quad , \quad T(X^-)=T(X^+)=T_{ev} \tag{12}$$

where $X^+ = \dfrac{x_{ev}^+}{l}$ and $X^- = \dfrac{x_{ev}^-}{l}$ are the non dimensional variables defining the drying zone.
This non linear eigenvalue problem may have several solutions, cf. [7]. But taking into account that $\lambda << 1$, one can look for a solution as a series expansion in λ. Thus, the solution takes the following form:

$$X^+ = x_0^+ + \lambda x_1^+ + \lambda^2 x_2^+ + ... \qquad X^- = x_0^- + \lambda x_1^- + \lambda^2 x_2^- + ... \tag{13a.b}$$

$$T = T_0 + \lambda T_1 + \lambda^2 T_2 + ... \qquad u = u_0 + \lambda u_1 + \lambda^2 u_2 + ... \tag{13c.d}$$

This calculation allows us to determine the evaporation zone and the rate of spread according to λ.
Let us notice that by the implicit function theorem, if such solution exists, it is unique in the vicinity of $\lambda = 0$. The detailed calculations of the expansion are given in [5].
The complexity of the expressions increases with the order of the approximation. We use a symbolic computation language to calculate the first seven orders of the series (13) which are given in the following table:

Table 1: Coefficients of the perturbation series related to *u*, X^- and X^+ related to the model I

orders	u	X^-	X^+
0	7.1518	2.4608	164.1169
1	- 0.4297	- 0.3200	- 10.1115
2	0.0165	0.0196	0.3922
3	0.0025	0.0057	0.0625
4	- 0.0182	4.5675e-004	- 0.4122
5	- 0.0144	- 0.0040	- 0.3300
6	0.0210	7.6727e-004	0.4770

The series (13) will converge to a limit only if λ lies within a circle whose radius is determined by the location of the nearest singularity in the complex λ-plane.We employ the method of Padé approximants to locate it. The calculation of the poles of these approximants allows us to give an approximated value of this singularity.
Herein we give poles of the sequence of Padé approximants [*n* + *j* , *n*], (*j* = -1, 0, 1) applied to the series (13.d)

Table 2: Poles of Padé approximants relative to the rate of spread

Padé table	Poles	Modules
P (5,1)	- 0.69182	0.69182
P (4,2)	0.1738 ± 0.7703i	0.7897
P (3,3)	0.1728 ± 0.7714i	0.7906
P (2,4)	0.1911 ± 0.7478i	0.7719
P (1,5)	- 0.8760	0.8760

The results show that there is a critical value close to $\lambda_0 = 0.77$, above which the series (13) are not convergent. This value coincides with the physical value $\lambda_c = 0.0587$.
It seems that this method is valid only for forest vegetation of thermal conductivity lower than λ_c.
One has to consider that for this flame model, the calculated value of the rate of spread is very close to the ones obtained experimentally and by identification [6].

4. CONCLUSION

A model of forest fire propagation including drying, pyrolysis, heat convection, heat conduction and radiative transfer is presented. This model has been obtained by asymptotic expansion and has the generic form of system of reaction diffusion equations. One of the aims of the present work was to link this type of reaction diffusion models to models relying on the concept of rate of spread. The rate of spread is defined as the solution of a non linear eigenvalue problem. This solution is obtained as a perturbation expansion in the heat conductivity of the solid phase. Calculated rate of spread for a flame model has been compared favourably with experimental values (obtained by thermocouples and by simple thermal sensor).

It has been demonstrated that the reaction-diffusion models have a possible ability for helping in the modelling of fighting. The leading effect for controlling the fire is the pyrolysis process. The influence of the activation energy must be studied, but other parameters of the pyrolysis law seem to have a great influence on propagation.

6. REFERENCES

1. A.M. Grishin, 1997, Mathematical modelling of forest fires and new methods of fighting them , Translated by M. Czuma, L. Chikina and L. Smokotina, ed. Frank Albini,

2. O. Séro-Guillaume, J. Margerit, 2002, Modelling forest fires. Part I: A complete set of equations derived by extended irreversible thermodynamics , *International Journal of Heat and Mass Transfer*, vol. 45 (8), 1705-1722

3. M. Larini, F. Giroud, B. Porterie, J.C. Loraud, 1998, A multiphase formulation for fire propagation in heterogeneous combustible media , *International Journal of Heat and Mass Transfer*, vol. 41, 881-897.

4. J. Margerit, O. Séro-Guillaume, 2002, Modelling forest fires. Part II: Reduction to two-dimensional models and simulation of propagation *International Journal of Heat and Mass Transfer*, Vol. 45 (8), 1723-1737

5. K Chetehouna., M. Er-Riani, O. Séro-Guillaume, 2004, On the rate of spread for some reaction diffusion models of forest fire propagation, *Numerical Heat Transfer, Part A*, Vol. 46, 765-784

6. K.Chetehouna, O. Séro-Guillaume, A. Degiovanni, 2005, Identification of the upward gas flow velocity and of the geometric characteristics of a flame with a specific thermal sensor, *International Journal of Thermal Sciences*, vol 47, pp 1148-1157

7. J. Xin, 2000, Front propagation in heterogeneous media, *SIAM Review*, vol. 42 (2), 161-230

8. J.C.S. Andre, M.J. Urbano, D.X. Viegas, 2006, Forest fire front spread models: The local quasi-equilibrium approach, *Combustion science and technology*, vol. 178, n°10-12, pp. 2115-2143

9. O. Séro-Guillaume, S .Ramezani, J. Margerit, D. Calogine, 2007, On large scale forest fires propagation model. *Int. J. Thermal Sci.* 47, 680-694

10. N.J. De Mestre, E.A. Catchpole, D.H. Anderson and R.C. Rothermel, 1989, Uniform propagation of a planar fire front without wind, *Combust. Sci. and Tech.*, vol. 65, pp. 231-244