A New Algorithm for Solving Transient Convective and Radiative Heat Transfer Problems in a Participating Medium

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Abstract

A transient double-population thermal lattice Boltzmann BGK scheme for the Natural Convection in the Presence of Volumetric Radiation is proposed to describe the heat and mass transfer in a two dimensional cavity containing an absorbing, emitting, and scattering medium. Heat radiation is solved using the control volume finite element method (CVFEM). Navier-Stokes equations, describing natural convection, are solved with a double population of a nine-velocity flow and temperature distributions in Lattice Boltzmann BGK Method. Where, separate particle distribution functions in the LBM are used to calculate the density, velocity fields and the thermal field. The effect of convection radiation parameter on Rayleigh Benard convection isotherm's is highlighted and discussed. The results have been compared with some available benchmark solutions and a good agreement has been observed.

Keywords: Rayleigh Benard convection; radiation; coupled LBM-CVFEM; particle distribution function (PDF), double population.

1. Introduction

The unsteady coupled free convection and radiation heat transfer has numerous applications in the area of fibrous and foam insulations, high temperature heat exchangers, industrial furnaces, optical textile fiber processing, thermal insulation, building comfort ,fire protection, combustors…[1-9]. In present paper, we carried out numerical investigation of combined natural convection in the presence of radiation in a rectangular cavity by considering perfectly conducting boundary condition for left and right walls. The bottom and top walls were maintained at hot and cold temperature respectively. The aim of this paper is to gain better understanding of heat transfer mechanism and fluid flow behavior for the case in hand. In order to do this, we carried out numerical investigation based on two approaches; the lattice Boltzmann [10-12] and control volume finite element methods [13-17]. .

This paper is arranged as follow. The physics and the boundary conditions of the problem are firstly defined, followed by an explanation of the mathematical and numerical methods. Then, the results are presented and discussed. The last section concludes current study.

2. Numerical models

The physical domain of the problem is represented in (Fig.1&2) where we consider a rectangular cavity bounded by two vertical insulating walls, and two horizontal isothermal walls at different temperature, T_h, and T_c, ($T_h > T_c$) respectively. The fluid in the cavity is emitting, absorbing, and isotropically scattering medium. Thermo-physical properties, except density, are assumed constant. Density is considered to vary in the Boussinesq sense. The boundaries of the cavity are considered diffuse and grey. In this study, the governing equation of incompressible, two-dimensional and laminar Navier-Stokes and energy equations were solved directly using an hybrid approach based on the lattice Boltzmann [10-12] and control volume finite element formulation [13-17]. .

2.1. Mathematical formulation

LBM starts with the Boltzmann equation, discretised in space and time, given as:

$$f_k(\vec{r}+\vec{c_k}\Delta t, t+\Delta t) = f_k(\vec{r},t)+\Omega_k+\Delta t\, F, \quad k=0,..,8 \tag{1}$$

where

$$\Omega_k = -\frac{\Delta t}{\tau_v}[\, f_k(\vec{r},t) - f_k^{\,eq}(\vec{r},t)] \tag{2}$$

$$f_k^{eq} = w_k \rho (1+\frac{c_k u}{c_s^2}+\frac{(c_k u)^2}{2c_s^4}-\frac{uu}{2c_s^2}) \tag{3}$$

f_k are the particle distribution function defined for the finite set of the discrete particle velocity vectors $\vec{c_k}$. The collision term Ω_k on the right-hand side of Eq. (1) uses the so called Bhatangar–Gross–Krook (BGK) approximation. $f_k^{\,eq}$ is the local equilibrium distribution function that has an appropriately prescribed functional dependence on the local hydrodynamic properties, τ_v is the relaxation time. F represents the external force.

For the D2Q9 lattice (Figure 1) used in the present work, the relaxation time τ_v is given by :

$$\tau_v = \frac{1}{2}+\frac{3\nu}{c^2\Delta t} \tag{5}$$

It is to be noted that viscosity ν is selected to insure that Mach number Ma is within the limit of incompressible flow [10-12].

The nine velocities $\vec{c}_k$ in the D2Q9 are depicted in figure 1.

The macroscopic density ρ and the velocity $\vec{u}$ are:

$$\rho(\vec{r},t)=\sum_k f_k(\vec{r},t) \tag{6.a}$$

$$u(\vec{r},t)=\sum_k c_k f_k(\vec{r},t)/\rho(\vec{r},t) \tag{6.b}$$

No slip boundary condition is imposed along all the walls for solving the hydrodynamic fields. In the LBM, the unknown particle distribution functions are directed into the medium from the wall. For the density distribution functions, bounce-back boundary conditions were applied on all solid boundaries, which mean that incoming boundary populations equal to out-going populations after the collision.

In the presence of volumetric radiation, the governing lattice Boltzmann equation for the thermal field, is given by [13-14, 16-17] :

$$g_k(x+\Delta x, y+\Delta y, t+\Delta t)=(1-(\frac{\Delta t}{\tau_T}))g_k(x,y,t)+(\frac{\Delta t}{\tau_T})\, g_k^{eq}(x,y,t)+S_{Rad} \tag{7.a}$$

where

$$g_k^{eq}=w_k T(1+\frac{c_k u}{c_s^2}) \tag{7.b}$$

where g_k is the particle distribution function denoting the evolution of the internal energy, τ_T is the relaxation time, g_k^{eq} is the equilibrium distribution function for thermal energy , S_{Rad} is the term source designed for simulate the radiative heat flux $\overrightarrow{q_R}$ .the temperature is given by:

$$T(\vec{r},t)=\sum_k g_k(\vec{r},t) \tag{8}$$

For the thermal boundary conditions, the two vertical walls are maintained at constant temperatures (T_h and T_c) and the two horizontal walls are adiabatic $\partial T/\partial y\big|_{(x,0)}=\partial T/\partial y\big|_{(x,H)}$.

So, to determine these isothermal temperatures, the normal equilibrium condition (Zu et He (1997)) was used.

For the adiabatic boundaries, the temperatures to be specified on the north boundary, for example, are expressed as:

$$g_k(x,n,t) = g_k(x,n-1,t), \quad k = 0,1..,8 \tag{9}$$

where n is the lattice at the boundary and $n-1$ is the lattice in the domain neighbor to the boundary lattice.

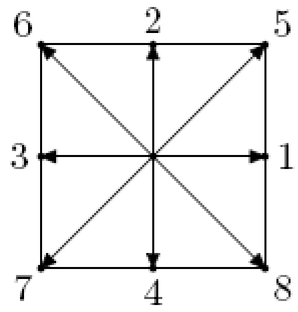

Fig. 1. Nine-velocity lattice model

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2.2. Radiative term

The radiative heat flux $\vec{q}_R$ and the divergence of radiative heat flux are given by:

$$\vec{q}_R = \int_{4\pi} I\vec{\Omega}d\Omega \tag{10.a}$$

$$\vec{\nabla}.\vec{q}_R = k_a(4\pi I_b - G) \tag{10.b}$$

I is the radiative intensity which can be obtained by solving the Radiative Transfer Equations (RTE). k_a is the absorption coefficient, $I_b = \sigma T^4/\pi$ is the blackbody intensity and G is the incident radiation.

The RTE for an absorbing, emitting and scattering grey medium can be written as:

$$\vec{\nabla}.(I(s,\vec{\Omega}).\vec{\Omega}) = -(k_a + k_d)I(s,\vec{\Omega}) + k_a I_b(s) + \frac{k_d}{4\pi}\int_{\Omega'=4\pi} I(s,\vec{\Omega'})\Phi(\vec{\Omega'} \to \vec{\Omega})d\Omega' \tag{11.a}$$

where $I(s,\vec{\Omega})$ is the radiative intensity, which is a function of position s and direction $\vec{\Omega}$; k_d is the scattering coefficient and $\Phi(\vec{\Omega'} - \vec{\Omega})$ is the scattering phase function from the incoming $\vec{\Omega'}$ direction to the outgoing direction $\vec{\Omega}$. The term on the left-hand side represents the gradient of the intensity in the direction $\vec{\Omega}$. The radiative boundary condition for Eq. (11.a), when the wall bounding the physical domain is assumed grey and emits and reflects diffusely, can be expressed as:

$$I_w(\vec{\Omega}) = \frac{\varepsilon_w \sigma T_w^4}{\pi} + \frac{1-\varepsilon_w}{\pi} \int_{\vec{\Omega}'.\vec{n}_w<0} I_w(\vec{\Omega}') \left| \vec{\Omega}'.\vec{n}_w \right| d\Omega' \quad \text{if} \quad \vec{\Omega}'.\vec{n}_w > 0 \tag{11.b}$$

$\vec{n}_w$ is the unit normal vector on the wall and ε_w represents the wall emissivity.

The control volume finite element method is used to discretize the RTE .In the CVFEM, the spatial and angular domains are divided into a finite number of control volumes and control solid angles, respectively. For angular discretization, the direction of propagation $\vec{\Omega}$ is defined by the couple(θ, φ) where θ and φ are, respectively, the polar and azimuthal angles.The total solid angle is subdivided into $N_\theta \times N_\varphi$ control solid angles as depicted in fig.2, where $\Delta\varphi = (\varphi^+ - \varphi^-) = 2\pi / N_\varphi$ and $\Delta\theta = \pi / \mathrm{N}_\theta$. The N_φ and N_θ represent numbers of control angles in the polar and azimuthal directions, respectively. These $N_\varphi N_\theta$ control solid angles are non-overlapping, and their sum is 4π . The control solid angle $\Delta\Omega^{mn}$ is given by (fig. 2):

$$\Delta\Omega^{mn} = \int_{\Delta\theta} \int_{\Delta\varphi} \sin\theta \, d\theta \, d\varphi \tag{12}$$

The spatial domain is subdivided into three-node triangular elements. As shown in fig. 2(b), a control volume ΔV_{ij} is created around each node N by enjoining the controids G_l of the elements to midpoints M_l and M_{l+1} of the corresponding sides. Each element has two faces, $M_l G_l$ and $G_l M_{l+1}$; bounding the sub-control volume around N; and each control volume is constructed by adding all subvolumes $NM_l G_l M_{l+1} N$, fig.2c. The obtained mesh is composed of $N_x N_y$ control volumes ΔV_{ij} .The N_x and N_y represent numbers of nodes in x and y direction, respectively. Δx and Δy represent the regular steps in x and y direction, fig. 1. After integrating the radiative transfer equation over both control volume and control solid, the final algebraic equation of the RTE is given by the following expression [37]:

$$\gamma_{1ij}^{mn} I_{ij-1}^{mn} + \gamma_{2ij}^{mn} I_{i+1j}^{mn} + \gamma_{3ij}^{mn} I_{i+1j+1}^{mn} + \sum_{(m',n')=(1,1)}^{(N_\theta,N_\varphi)} \alpha_{ij}^{mnm'n'} I_{ij}^{m'n'} + \gamma_{4ij}^{mn} I_{ij+1}^{mn} + \gamma_{5ij}^{mn} I_{i-1j}^{mn} + \gamma_{6ij}^{mn} I_{i-1j-1}^{mn} = \beta_{ij}^{mn} \tag{13}$$

Then, the algebraic eq. (13) is written in an adequate matrix form [39-40]. The obtained equation set is solved using the conditioned conjugate gradient squared method (CCGS). A detailed on calculation can be found in ref. [13-17].

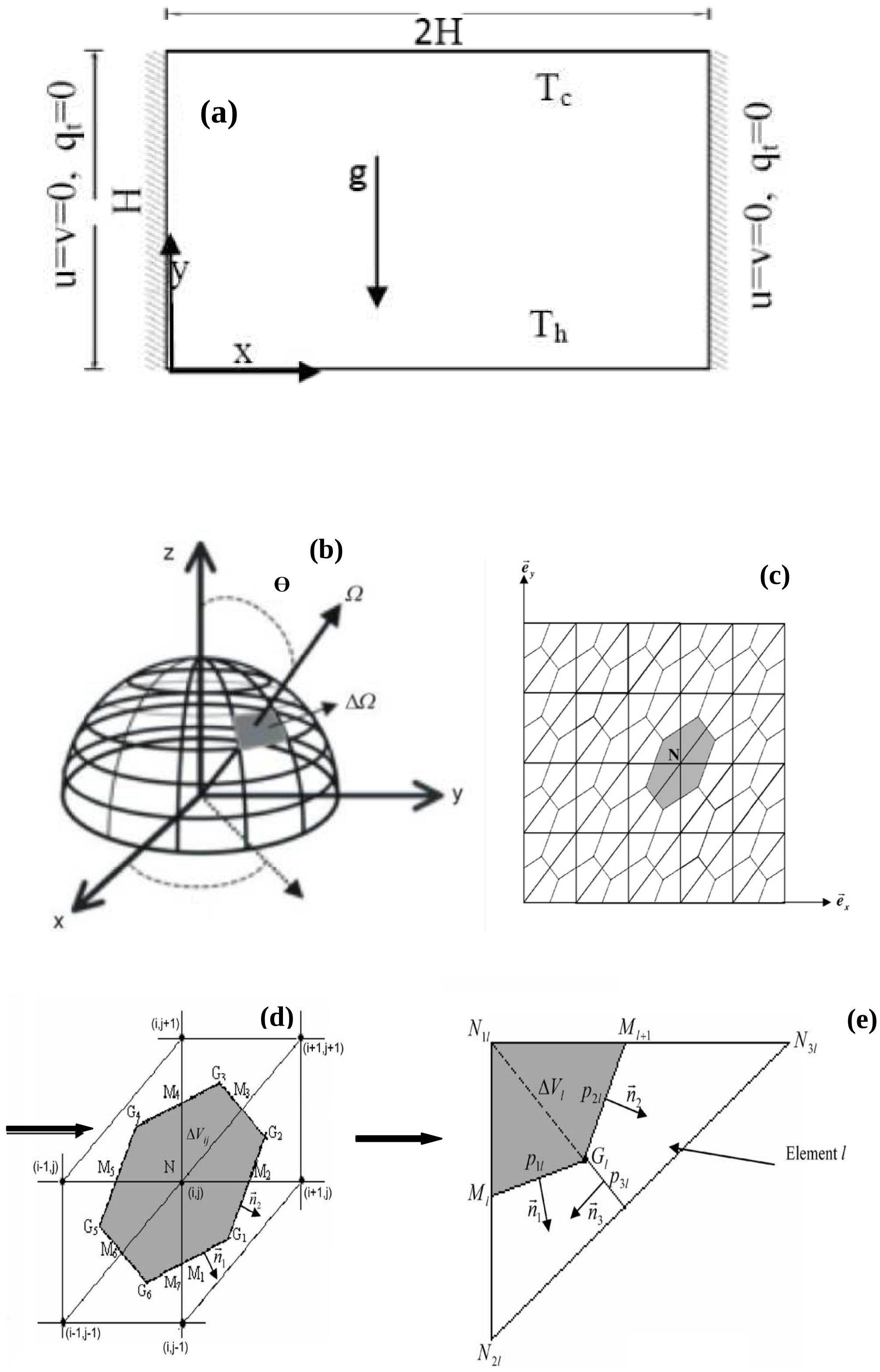
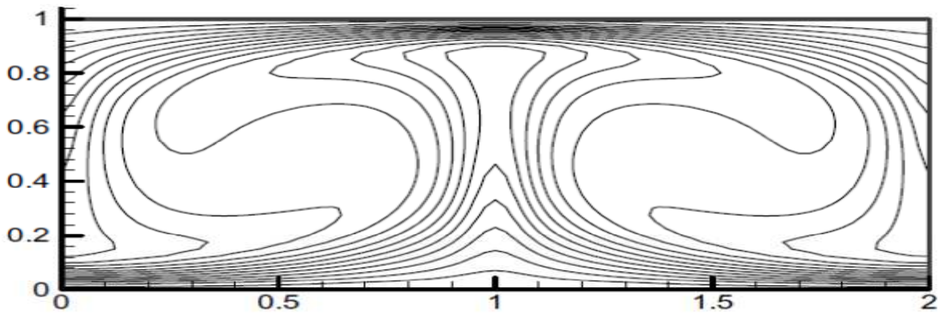


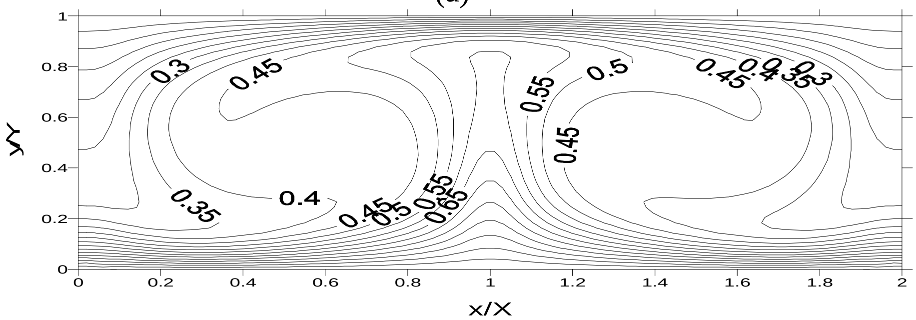
Fig. 2 (a) Geometry of the cavity, (b) Angular and spatial discretization in ($\vec{e_x}, \vec{e_y}$) plan, (c) Control volume ΔV_{ij}, (d) subvolume δV_{ij}.

3. Results

The numerical procedure is validated with the steady state Rayleigh Benard convection generated numerically for differentially heated rectangular cavity by Kao et al. [18]. Validation of the numerical code to predict the isotherms profiles was performed at first in a differentially heated cavity without radiation by comparing the results published in Kao et al. [18]. Figure 3 shows the results for a two dimensional Rayleigh–Benard convection simulation that considered the case of $\mathrm{Pr} = 0.71$, this figure presents the corresponding temperature contours at $Ra = 10^5$. The numerical procedure is found to be satisfactory with this Rayleigh number and the new numerical algorithm based on LBM show very good agreement with the existing numerical results.



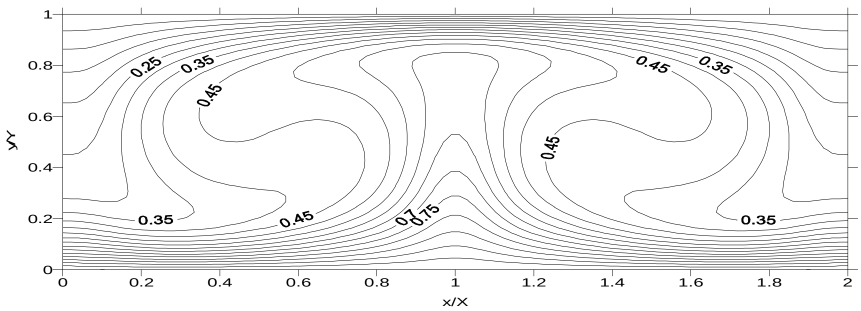
(a)



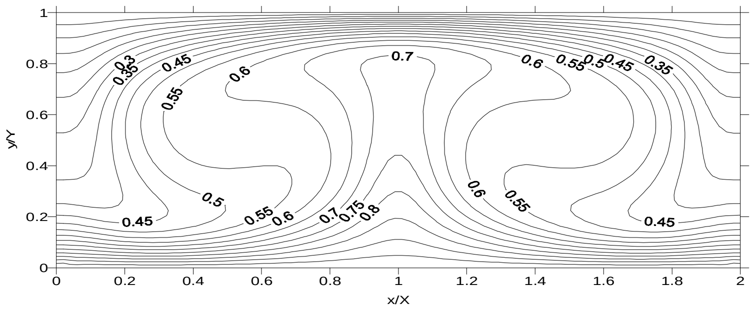
(b)

Figure 3: Temperature contours at $Ra = 10^5$ and $\Pr = 0.71$ (a) reference [18], (b) present work.

The transient Benard convection with radiation spanning an aspect ratio AR=2 is treated. The problem under consideration involves the effect of the convection radiation parameter RC on temperature profiles. Figure 4 show the evolution of the isotherms that present a significant change as the convection radiation parameter RC increase and become almost parallel to the hot and cold walls. We notice that dense isotherms occur in a region close to the horizontal cold wall, indicating a high temperature gradient there. So, as the convection radiation parameter RC increases, the effect of radiation increases and the flow is therefore stabilized by the presence of the radiative source.



(a)



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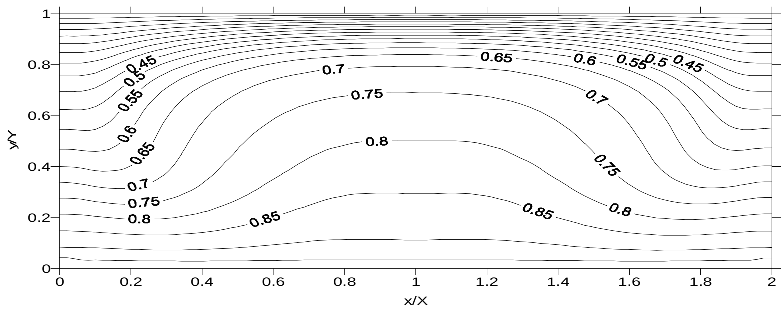
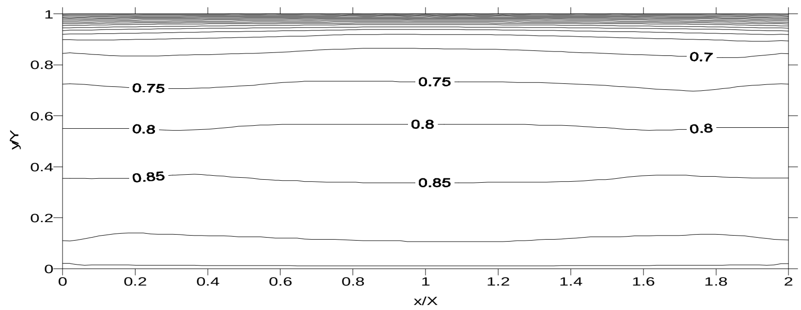


(c)



(d)

Figure 4: Thermal field predicted by LBGK model for $Ra = 5.10^4$ (a) in the absence of radiation, (b) $RC = 0.1$, (c) $RC = 1.0$ and (d) $RC = 10.0$.

3. Conclusions

LBM-CVFEM for combined transient convection and radiation systems has been explored. The unsteady laminar Raleigh Benard convection in the presence of volumetric radiation in a rectangular cavity containing an absorbing, emitting and scattering medium is considered. Without the effect of radiation the results for natural convection were compared with those available in literature. The results were found in good agreements. It has been shown that the new hybrid algorithm reproduce all the known features of the coupled convection-radiation phenomena in the participating media.

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given by [31] :

$$F = \left[\frac{(\rho \beta_T g (T - T_m) \vec{j}).(\overrightarrow{c_k} - \vec{u})}{RT} \right] f_k^{eq} \tag{4}$$

where the unit vector $\vec{j}$ is in a direction opposite to gravity, T_m is the mean temperature, g is the gravity acceleration, β_T is the volumetric thermal expansion coefficient and ρ is the density of the fluid at the mean temperature T_m .