NUMERICAL STUDY OF NATURAL CONVECTION IN A FLUID-SATURATED POROUS MEDIA IN AN ELLIPTICAL ANNULUS WITH A VARIABLE ORIENTATION

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ABSTRACT

Two-dimensional natural convective flow in a tilted elliptical annulus porous material saturated with fluid is analyzed by solving numerically the mass, momentum and energy balance equations, using Darcy's law and the Boussinesq approximation. Isothermal boundary conditions are considered, where the walls delimiting the annular space are maintained at two uniform different temperatures. The external parameters considered are the tilt angle and the modified Rayleigh-Darcy number. Two main convective modes are found: double and multiple cell convection and their features described in detail. Local and average Nusselt numbers are presented in terms of external parameters.

Keywords : natural convection; elliptic cylinders; Rayleigh number; Darcy number; isotherms

INTRODUCTION

During recent years a considerable research effort has been devoted to the study of heat transfer induced by natural convection in a porous medium saturated by a fluid. Interest in these phenomena of natural convection is due to many potential applications in engineering. These applications include the extraction of geothermal energy, the dispersion of pollutants in aquifers, security issues in the heart of nuclear reactors, thermal insulation of buildings…etc.

[The effect of the porous sleeve on the buoyancy induced flow motion under steady-state condition was examined. [Kumari and Nath (2008)] studied the unsteady natural convection flow from a horizontal cylindrical annulus filled with a non-Darcy porous medium .The unsteadiness in the problem arises due to the impulsive change in the wall temperature of the outer cylinder. The Navier–Stokes equations along with the energy equation governing the unsteady natural convection flow have been solved by the finite-volume method. The results show that the annulus completely filled with a porous medium has the best insulating effectiveness. The effect of Darcy number on the heat transfer is more pronounced than that of the Grashof number. [Charrier-Mojtabi (1997)] carried a numerical investigation of two-dimensional and three-dimensional free convection flows in a saturated porous horizontal annulus heated from the inner surface, using a Fourier-Galerkin approximation for the periodic azimuthal and axial directions and a collocation-Chebyshev approximation in the confined radial direction. The numerical algorithm integrates the Darcy-Boussinesq's equations formulated in terms of pressure and temperature. This method gives an accurate description of the 2-D multicellular structures for a large range of Rayleigh number and radii ratio. Bifurcation points between 2-D unicellular flows and either 2-D multicellular or 3-D flows are also determined numerically. [Sankar *et al* (2011)] investigated natural convection flows in a vertical annulus filled with a fluid-saturated porous medium, when the inner wall is subject to discrete heating. The outer wall is maintained isothermally at a lower temperature, while the top and bottom walls, and the unheated portions of the inner wall are kept adiabatic. Through the Brinkman-extended Darcy equation, the relative importance of discrete heating on natural convection in the porous annulus is examined. An implicit finite difference method has been used to solve the governing equations of the flow system. The analysis is carried out for a wide range of modified Rayleigh and Darcy numbers for different heat source lengths and locations. The numerical results reveal that an increase in the radius ratio, modified Rayleigh number and Darcy number increases the heat transfer, while the heat transfer decreases with an increase in the length of the heater. [Ching-Yang C. (2007)] used a thermal non-equilibrium model to study the free convection boundary layer flow driven by temperature gradients near a permeable horizontal cylinder of elliptic cross-section with constant wall temperature in a fluid-saturated porous medium. A coordinate transformation is used to obtain the nonsimilar boundary layer equations. The transformed boundary layer equations are then solved by the cubic spline collocation method. Results for the local Nusselt numbers are presented as functions of the porosity scaled thermal conductivity ratio, the heat transfer coefficient between solid and fluid phases, the transpiration parameter, and the aspect ratio when the major axis of the elliptical cylinder is vertical (slender orientation) and horizontal (blunt orientation). An increase in the porosity scaled thermal conductivity ratio or the heat transfer coefficient between the solid and fluid phases increases the heat transfer rates. Moreover, the use of suction (positive transpiration parameter) tends to increase the heat transfer rates between the porous medium and the surface.

RESULTS AND DISCUSSION

For the validation of the computational problem, we compared our results with those of literature: The numerical simulation of two- and three-dimensional free convection flows in a horizontal porous annulus using a pressure and temperature formulation of [Charrier-Mojtabi (1997)] and the natural convection heat transfer in horizontal eccentric elliptic annuli containing saturated porous media of [Mota *et al.* (2000)].
The grid dependence has been investigated using different mesh sizes before settling to a mesh size of (61x 81).
Our objective is to analyze the effect of tilt on heat transfer and flow. For this reason, we presented the isotherms and streamlines in different inclinations for different modified Rayleigh-Darcy numbers for a space annulus delimited by two elliptic cylinders whose respective eccentricities are e_1=0.9 and e_2=0.75.

Influence of the modified Rayleigh-Darcy number

The figures (3-7) represent the isotherms and streamlines for different values of modified Rayleigh-Darcy number Ra_m when α=0°.
We note that these isotherms and these streamlines are symmetrical about the median fictitious vertical plane.
The isotherms of figure (3) corresponding to Ra_m =100 are parallel and concentric closed curves which coincide well with the wall profiles, in this case the temperature distribution is simply decreasing from the hot wall to the cold wall. The streamlines of the same figure show that the flow is organized in two main cells that rotate very slowly in opposite directions. This is due to upward movement of the fluid particles which heat up along the hot wall under the effect of buoyancy and the downward movement of the fluid particles which cool along the cold wall under the gravity. The values of the Stream function are very low. In this case heat transfer takes mainly by conduction at the heated wall, although the velocity fields are nonzero.

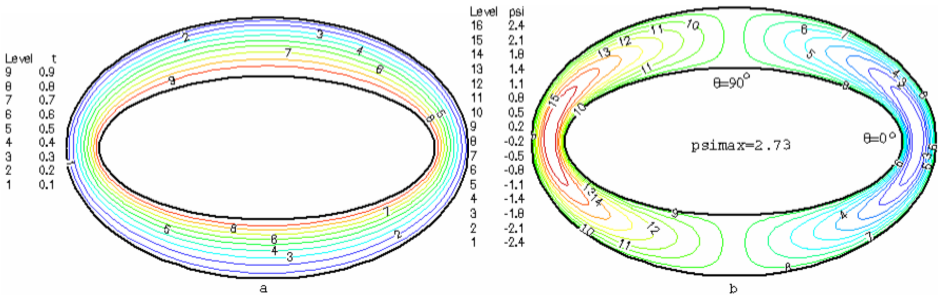

Fig. 3. Isotherms and Streamlines for Ra_m=100 and α=0°.

Figures (4) and (5) corresponding to Ra_m=135 and Ra_m=137 show that the isotherms are modified. The temperature distribution decreases from the hot wall to the cold wall. The deformation direction of the isotherms is conforming to the direction of streamlines rotation. Values of streams function are increased which means that the convection intensifies.

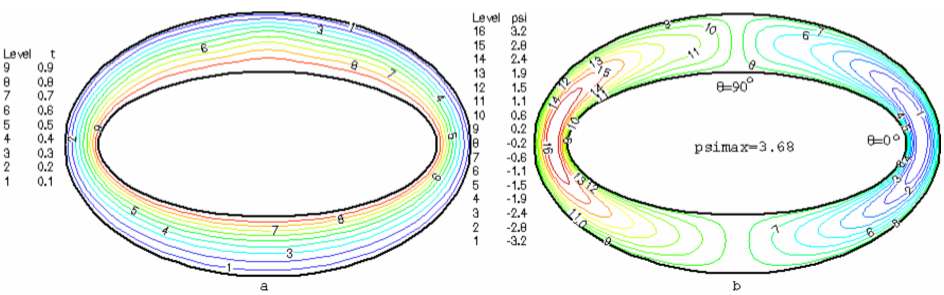

Fig. 4. Isotherms and Streamlines for Ra_m=135 and α=0°.

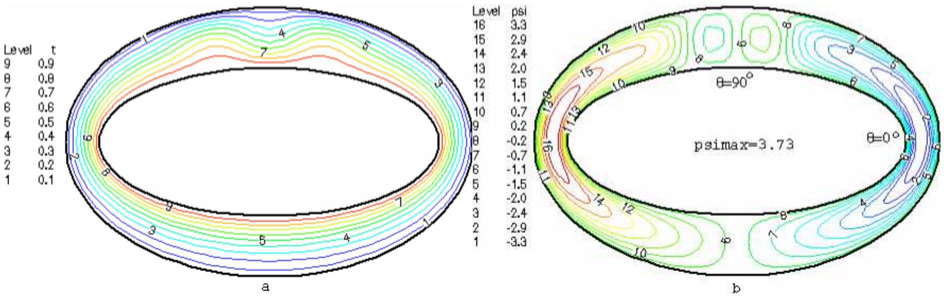

Fig. 5. Isotherms and Streamlines for Ra_m=137 and α=0°.

In figure (6) for Ra_m=150, increasing the Darcy modified Rayleigh number reflects an intensification of natural convection, this increase has allowed the appearance of a bifurcation giving rise to two additional cells turning in the otherwise direction of neighbor cells.

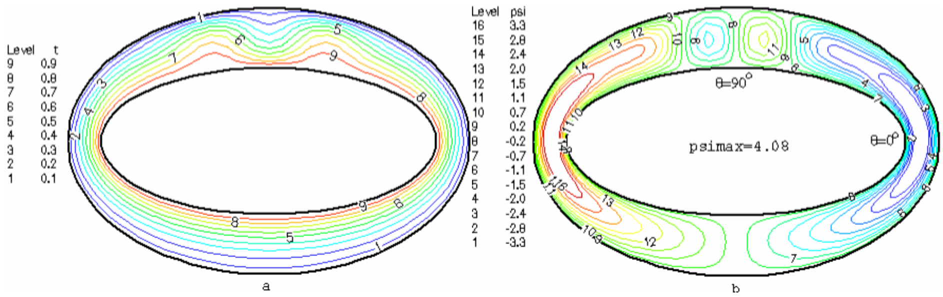

Fig. 6. Isotherms and Streamlines for Ra_m=150 and α=0°.

The figure (7) for Ra_m=500 shows that we have two bifurcations with four additional cells each one turns in the opposite direction of its neighbor cell. The increase in the modified Rayleigh-Darcy number has allowed passing to another flow regime that is the multicellular flow, with the appearance of these bifurcations in the upper part of the annulus which represent a zone of instabilities for large values of the modified Rayleigh-Darcy number. Isotherms of these figures show that the fluid is almost motionless in the bottom of the enclosure and in the top they deform and sink into the area where there is presence of two counter-rotating vortices.

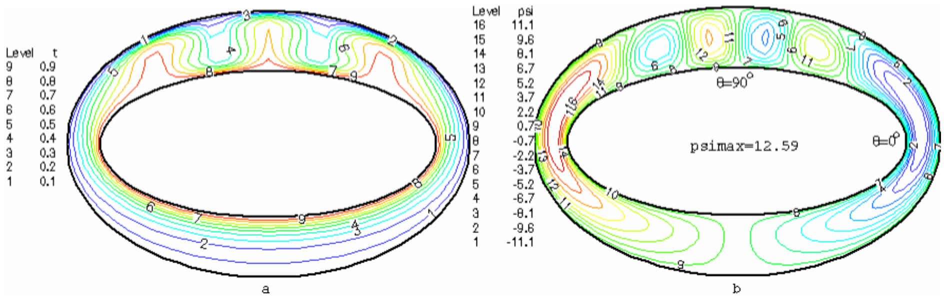

Fig. 7. Isotherms and Streamlines for Ra_m=500 and α=0°.

The variation of local Nusselt number on the inside elliptical cylinder

Figure (8) shows the variation of local Nusselt number on the inside wall of the elliptical cylinder, and allows us to note that with the increase of modified Rayleigh-Darcy number, the value of local Nusselt number increases, which is obvious. This figure shows for Ra_m=500, the existence of three minimums and two maximums in the top of the enclosure (30°<θ<150°) corresponding to the juxtaposition of two counter-rotating cells pushing away the fluid from the hot wall in the case of a minimum, and bringing the fluid to the wall in the case of a maximum.

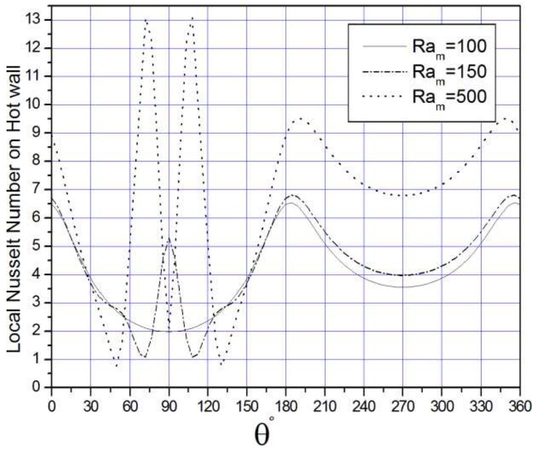

Fig. 8. Variation of local Nusselt number on the hot wall for α=0°.

Influence of the variation of the inclination angle α

In this section we examine the effect of the inclination of the system. The angle is calculated from the horizontal in the trigonometric direction. To get this, we have five values of α (0°, 10°, 45°, 60° and 90°).

Case where the inclination angle is zero

In this case, the vertical fictitious median is in principle a symmetry plane for transfer phenomena. Therefore by symmetry and in relation to this vertical plane depending on the value of modified Rayleigh-Darcy number, the flow is organized in two principal cells rotating in opposite directions or with two or four cells in addition depending on the case considered, as the figures (3-7) show.

Case where the inclination angle α≠0° and α≠90°

By giving the angle any value other than 0 ° and 90 °, we destroy the symmetry of the system relative to the fictitious vertical plane. The figures (9) and (10) respectively for α= 10° and α= 45 ° with Ra_m=500 show that the vortex can be further developed like its counterpart on the left side. The lower right cell also develops like its counterpart on the left side which moves to the upper part of the annular space to merge with the upper vortex of the right part of the system. By increasing the inclination angle to reach α= 60 °, Figure (11) for Ra_m=500 shows that these two vortices when developed occupy the entire annular space.

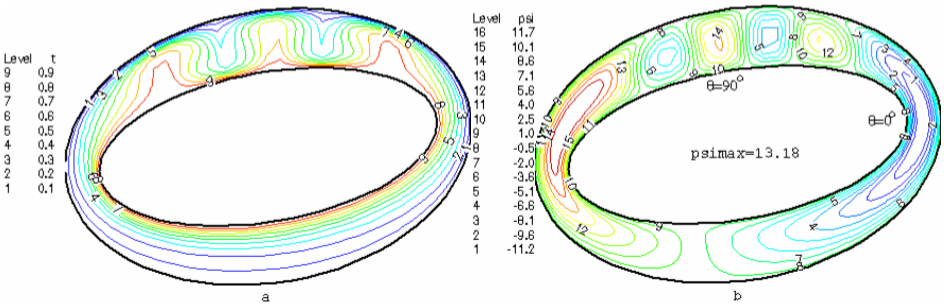

Fig. 9. Isotherms and Streamlines for Ra_m=500 and α=10°.

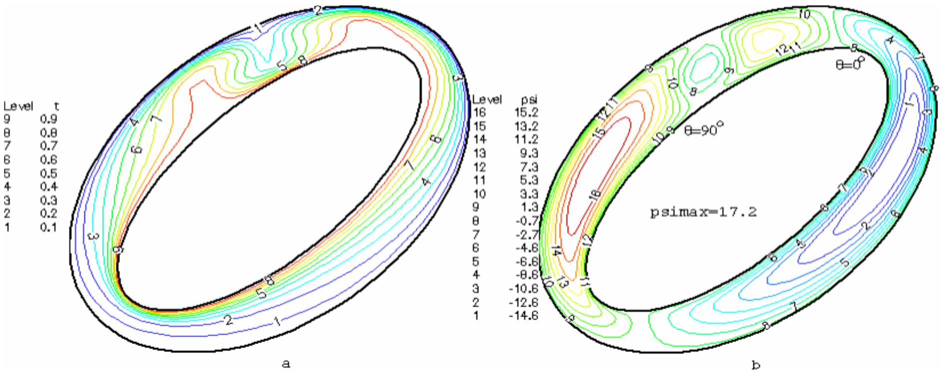

Fig. 10. Isotherms and Streamlines for Ra_m=500 and α=45°.

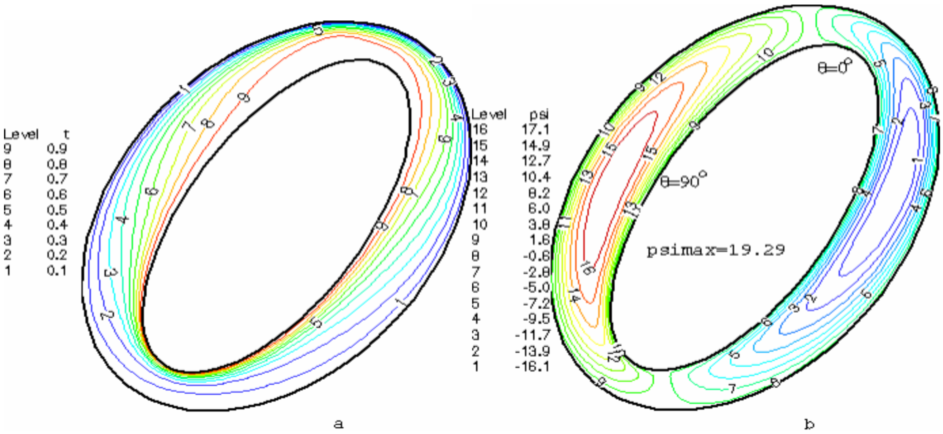

Fig. 11. Isotherms and Streamlines for Ra_m=500 and α=60°.

Case where the inclination angle α=90°

When α= 90°, the vertical fictitious plane passing through the center of the system is again a plane of symmetry. Figure (12) shows that the isotherms are closed curves almost parallel and concentric that coincides well with the wall profiles, in the summital region, these isotherms fit upward, this is due to separation of particles moving away from the hot wall at the plane of symmetry, thus showing the presence on both sides of the latter, two vortices taking the fluid from the hot wall to the cold wall. The temperature decreases from the hot lower wall to the cold upper wall.

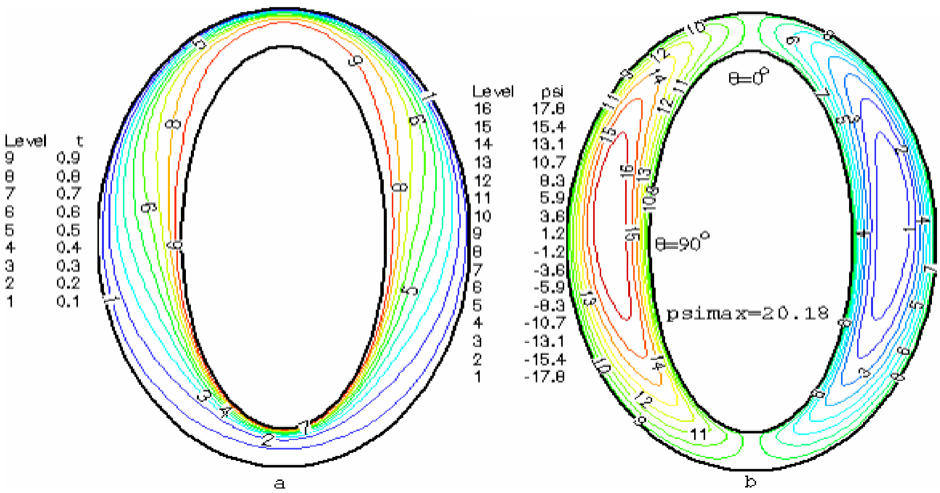

Fig. 12. Isotherms and Streamlines for Ra_m=500 and α=90°.

CONCLUSION

The phenomenon of natural convection in a porous elliptical annulus that is saturated of Newtonian fluid was studied using a numerical method. The bicellular or multicellular convection takes place according to the value of the modified Rayleigh-Darcy number and the angle of inclination.
For the inclination angle $\alpha=0$ ° the main mode of heat transfer is the multi-cellular convection for high Rayleigh-Darcy numbers. For $\alpha\neq0°$, the system behavior of the natural convection is completely different when the system is tilted; both modes of convection are present. With inclination angles<60 ° for $Ra_m=500$ the preferred mode of circulation is multicellular mode, whereas for $60°\leq\alpha\leq90°$, the preferred mode is bicellular.

NOMENCLATURE

a	Thermal diffusivity, $m^2.s^{-1}$
A	Elliptic cylinder major axis, m
B	Elliptic cylinder minor axis, m
c	constant defined in the elliptic coordinates, m
c_p	Specific heat at constant pressure, $j.kg^{-1}.K^{-1}$
Da	Darcy number
e	Elliptic cylinder eccentricity, $\left[=\sqrt{\frac{(A^2-B^2)}{A^2}}\right]$
g	Gravitational acceleration, $m.s^{-2}$
Gr	Grashof number, $[=g\beta c^3(T_1-T_2)/\upsilon^{-2}]$
h	Dimensional metric coefficient, m
H	Dimensionless metric coefficient
K	Porous medium permeability, m^2
Nu	Local Nusselt number
P	Pressure, $N.m^{-2}$
Pr	Prandtl number, $[=(\upsilon\rho c_p)/\lambda]$
Ra	Rayleigh number, $[=Gr.Pr]$
Ra_m	Modified Rayleigh-Darcy number, $[=Ra.Da]$

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