# Thermosolutal convection in a tilted square fluid enclosure subject to cross fluxes of heat and solute

**MEBROUKI Abdelhakim[1], MAMOU Mahmoud[2], SAIM Rachid[1]**
[1]*Energetic and Applied Thermal Laboratory (ETAP), Technology Faculty,*
*Abou Bakr Belkaid University, Tlemcen, Algeria.*
[2]*Aerodynamics Laboratory, National Research Council, Ottawa, Ontario,Canada.*
*Emails: abdelhakim.mebrouki@mail.univ-tlemcen.dz, Mahmoud.Mamou@nrc.ca, r_saim@mail.univ-tlemcen.dz*

**Abstract:** The present study dealt with a numerical study of double diffusive natural convection flow in a binary fluid contained in a tilted square cavity under cross temperature and concentration gradients. The mass, momentum and energy conservation equations were solved numerically using a finite-difference method. The study was focused on the cavity tilted at an angle of 45°, and when the thermal and solutal buoyancy forces were equal. The Results are presented in terms of the averaged Nusselt and Sherwood numbers, and the flow intensity as functions of the thermal Rayleigh and the Lewis numbers. The existence of the onset of convection was demonstrated and both natural and anti-natural flows solutions were obtained. Also, when the Lewis number is bigger or smaller than unity, subcritical flows are found to exist for natural convective solutions. The critical values of the thermal Rayleigh number for the onset of supercritical and subcritical convection were obtained.

**Keywords:** Square tilted cavity, Thermosolutal convection, Cross gradients of temperature and concentration, Numerical study.

## 1.Introduction

Double diffusive natural convection phenomenon in a confined fluid enclosure has received considerable attention among researchers and scientists owing to its practicality importance in geophysics and many engineering processes and applications. The groundwater contamination, melting and solidification of binary alloys, migration of moisture in fibrous insulation, chemical reactors, and drying processes are some examples where thermosolutal convection is a common occurrence.

In this paper, the problem of Paliwal and Chen [1] was considered to examine the effect of space confinement on double diffusive convection within a tilted enclosure. The authors had performed experimental investigation within a tilted slender slot subject to cross-gradient of temperature and solute. Positive angle denoted heating the lower wall, while negative angle denoted heating the upper wall. The temperature difference across the slot was increased progressively until convective instability was triggered. Flow patterns visualization was performed using a shadowgraph technology. The critical thermal Rayleigh number for the onset of instability was found to be non-symmetrical with respect to the vertical position. The heating from the lower wall was less stable. Secondary convective flows consisting of horizontal convective layers were found to be stable when $\theta < 0°$ because of the stabilizing temperature gradient.

In a second part of their study, Paliwal and Chen [2] applied a linear stability analysis of the basic convective flow when the slot was filled with a density-stratified fluid subject to a lateral temperature gradient. The derived stability equations were solved using the Galerkin technique. Within the range of $\theta$ considered in the experimental investigation [1], instability was found to be of a stationary type. The predicted results for the critical thermal Rayleigh and wave numbers at all inclination angles were found in good agreement with the experimental data. Contrary to the expected occurrence, the results showed that the system is more stable when the lower wall is heated. That was caused by the increased vertical solute gradient in the steady state prior to the onset of instabilities when the heating is from bellow.

In the present study, we examine the confinement effect on double diffusive convection within an inclined square fluid layer subject cross fluxes of heat and solute. The interest was focused on the effect of the thermal Rayleigh number, $Ra_T$, and the Lewis number, $Le$, on the heat and mass transfer rates for the situation where the thermal and solutal buoyancy forces were equal but were in opposite direction horizontally ($N$=1). A numerical procedure based on a second-order finite difference approach was considered to solve the full governing equations. The existence of subcritical and supercritical convection was demonstrated and the corresponding thresholds were determined.

## 2. Mathematical Formulation

The configuration considered in the present study is an inclined square cavity filled with a binary fluid mixture. The origin of the coordinate system is located at the center of the cavity, as shown in Fig. 1. Two of its parallel walls were subject to constant heat fluxes, *q'*, and are impermeable to mass, the other walls were subject to constant mass fluxes, *j'*, but kept adiabatic. The fluid was assumed to be Newtonian and obeying the Boussinesq approximation.

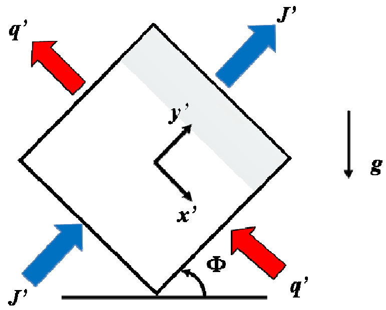


Figure 1: Flow configuration geometry and coordinates system.

The governing equations that govern the double-diffusive convection were expressed in terms of the continuity, momentum, energy and solute concentration conservation equations. They were given in dimensionless form using a stream function-vorticity $(\psi, \omega)$ formulation:

$$\frac{\partial \nabla^2\psi}{\partial t} - \frac{\partial\psi}{\partial x}\frac{\partial \nabla^2\psi}{\partial y} + \frac{\partial\psi}{\partial y}\frac{\partial \nabla^2\psi}{\partial x} = Pr\nabla^4\psi - Pr\,Ra_T\left(\sin\Phi\frac{\partial(T+N\,S)}{\partial x} + \cos\Phi\frac{\partial(T+N\,S)}{\partial y}\right) \quad (1)$$

$$\nabla^2\psi = -\omega \quad (2)$$

$$\nabla^2 T = \frac{\partial T}{\partial t} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y} + \frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} \quad (3)$$

$$\nabla^2 S = Le\left(\frac{\partial S}{\partial t} - \frac{\partial\psi}{\partial x}\frac{\partial S}{\partial y} + \frac{\partial\psi}{\partial y}\frac{\partial S}{\partial x}\right) \quad (4)$$

$$u = \frac{\partial\psi}{\partial y}\ ; \quad v = -\frac{\partial\psi}{\partial x} \quad (5)$$

where $Ra_T$, $Le$ and $Pr$ are the thermal Rayleigh, Lewis and Prandtl numbers, respectively, and $N$ is the buoyancy ratio. These parameters are defined in the nomenclature.

The associated dimensionless boundary conditions are:

$$\left.\begin{array}{llll} y = \pm\frac{1}{2} & \psi = \frac{\partial\psi}{\partial x} = 0, & \frac{\partial T}{\partial y} = 0 & \frac{\partial S}{\partial y} = -1 \\ x = \pm\frac{1}{2} & \psi = \frac{\partial\psi}{\partial y} = 0, & \frac{\partial T}{\partial x} = 1 & \frac{\partial S}{\partial x} = 0 \end{array}\right\} \quad (6)$$

where $\omega$, $\psi$, $T$ and $S$ are the dimensionless vorticity, stream function, temperature and solute concentration, respectively.

The average Nusselt and Sherwood numbers characterizing the heat and mass transfer rates are given respectively by:

$$Nu_m = \int_{-\frac{1}{2}}^{+\frac{1}{2}} \frac{1}{T_{\left(\frac{1}{2},y\right)} - T_{\left(-\frac{1}{2},y\right)}}\,dy \quad \text{and} \quad Sh_m = \int_{-\frac{1}{2}}^{+\frac{1}{2}} \frac{1}{S_{\left(x,-\frac{1}{2}\right)} - S_{\left(x,\frac{1}{2}\right)}}\,dx \quad (7)$$

## 3. Numerical solution

A standard numerical method based on a second-order finite difference approach was used to solve the governing equations. An alternating direction implicit (ADI) method was used to solve the discretized transport equations. However, the stream function equation (2) was solved using the point successive over-relaxation method (SOR). The vorticity at the walls was discretized using Jensen method [4]. A uniform grid was used in the two directions. The numerical code was validated with the results of de Val Davis [3] and Mamou *et al* [5] with satisfactory accuracy.

## 4. Results and discussion

In the present paper, the situation where the buoyancy ratio $N$=1 and the enclosure tilt angle $\Phi$ = 45° was considered. The effect of the Rayleigh and Lewis numbers on the flow behavior and on the heat and mass transfer rates were investigated, and the thresholds for the onset of infinitesimal and finite amplitude convection were determined.

The situation corresponding to $N$=1 and $\Phi$ = 45° could result in stable motionless state, which became instable above certain critical Rayleigh numbers. In the motionless state, there was a vertical density stratification within the enclosure, as the horizontal components of the thermal and solutal buoyancy forces were equal but opposing each other. This situation was studied experimentally and theoretically in the past by Paliwal and Chen [1]-[2] in a tilted slender fluid layer. Overall, there existed a supercritical Rayleigh number for the onset of convection, below which subcritical convection existed when the thermal and solutal diffusivities were not equal ($Le \neq 1$). Obviously, the thresholds for the onset of supercritical or subcritical convection depended on the Lewis number. For moderate values of *Le*, natural and anti-natural convective solutions co-existed.

Above the onset of supercritical convection, Fig. 2 displays the streamlines, isotherms and solute iso-concentrations, for $Ra_T = 10^5$ and $Le$=1. For this value of the Lewis number, as shown in the figure, the natural and anti-natural convective cells are identical but circulating in opposite direction. As the thermal and solutal diffusivities were equal, the two solutions had the same occurrence potential. For the solution where the cell was counter-clockwise, the convective flow was driven by the thermal effect and by solutal effect when it was clockwise. Owing to this driving effect, the solution led to different heat and mass transfer rates. The natural convective solution was defined for the fact it prevailed when initiating the convective flow from a motionless state. In this regards, when the Lewis number was bigger than unity, the natural solution was driven by thermal effects and the convective cell was counter-clockwise, as the thermal diffusivity effect prevailed. When the Lewis number was smaller than unity, the natural solution was driven by solutal effects and the convective cell circulation was clockwise. On the other hand, the anti-natural solution was obtained by forcing the flow in the opposite direction. Usually, at moderate Lewis number, the anti-natural could be sustained for a wide range of the Rayleigh number. However, it became unstable when it was approaching the threshold for supercritical convection and a jump to the natural convective solution may occur prematurely.

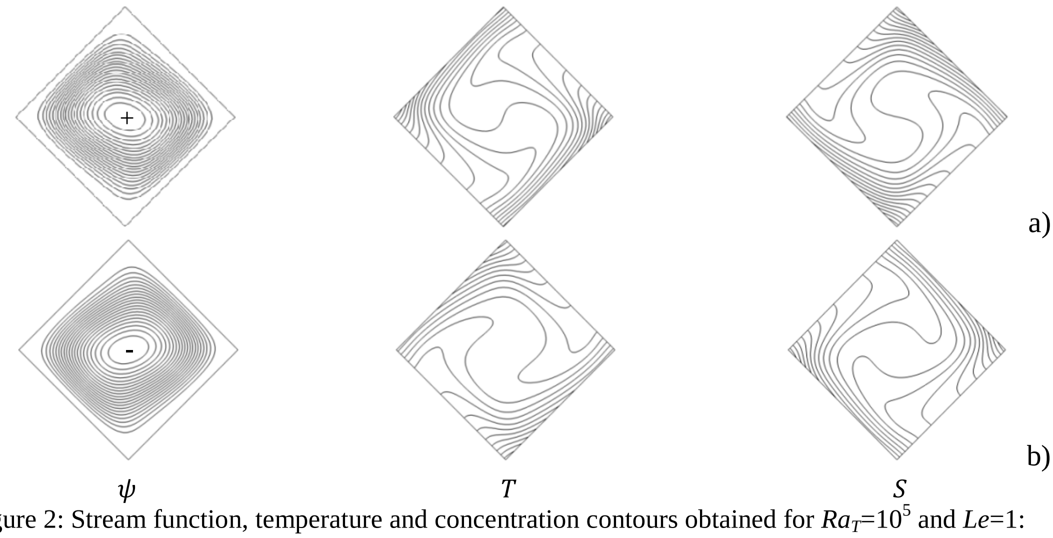

Figure 2: Stream function, temperature and concentration contours obtained for $Ra_T$=$10^5$ and $Le$=1: a) $\psi_0$= 15.767, $Nu_m$= 4.197, $Sh_m$ = 3.574, and b) $\psi_0$= -15.767, $Nu_m$= 3.574, $Sh_m$ = 4.197.

To investigate the convective behavior of the natural and anti-natural solutions, the flow intensity, $\psi_0$, and the averaged heat and mass transfer rates, $Nu_m$ and $Sh_m$, are givens in Fig. 3 as functions of $Ra_T$ and different values of *Le*. Regardless the value of *Le*, both natural and anti-natural solutions showed an increase of $|\psi_0|$, $Nu_m$ and $Sh_m$ when $Ra_T$ was increased. Concerning, the Lewis effect, the figure shows that $|\psi_0|$ decreased and $Sh_m$ increased when *Le* was increased for both natural and anti-natural solutions. However, $Nu_m$ of the anti-natural solution was decreased with *Le* increase. Far from the onset of convection, $Nu_m$ of the natural solution seemed not significantly affected by the *Le* variation, but near criticality, the threshold of the onset of convection increased with decreasing *Le*.

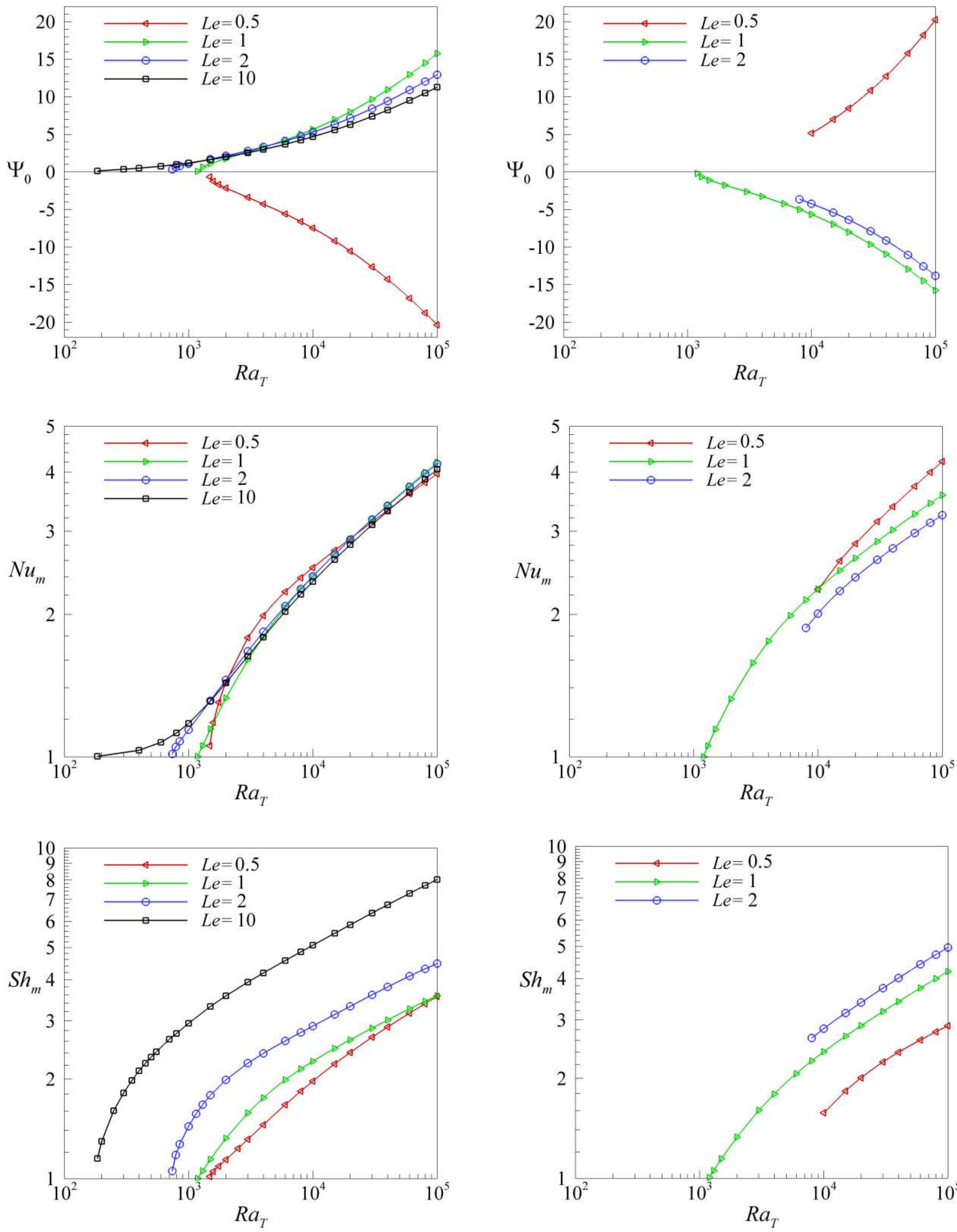


Figure 3: Natural (left) and anti-natural (right) solution bifurcation diagrams: flow intensity, $\psi_0$ , heat and mass transfer rates, $Nu_m$ and $Sh_m$, as functions of the Rayleigh number, $Ra_T$, for various Lewis number values, *Le*.

As explained earlier, both natural and anti-natural convective solutions bifurcated from the rest state solution at a given critical Rayleigh number. The threshold of supercritical convection was obtained accurately using a linear analysis by marching the solution in time for extremely weak convective flows, using the numerical solution of the full governing equations. The threshold of subcritical convection and the onset of anti-natural convective were approximately determined from the numerical solution at finite amplitude convection. First, the determination of the supercritical Rayleigh number is explained.

As known, for infinitesimal amplitude convection, the time evolution of the flow intensity is exponential, according to the linear stability analysis. Thus the flow intensity could be expressed as $\psi_0 = qe^{pt}$ , where $q$ is the amplitude at $t$=0 ($i.e. \psi_0$=$q$ at $t$=0). The parameter $p$ represents the amplitude growth rate. Typically, the value of $\psi_0$ within the range of $10^{-6} < \psi_0 < 10^{-4}$ was considered and judged small enough to assume infinitesimal amplitude. When p<0 the flow was decaying and when $p$>0 the flow was amplified. After knowing approximately the location of the threshold number, using the fully nonlinear solution, the solution was computed for two values of the Rayleigh number; one above and one below the threshold. Above the threshold, the numerical solution was marched in time from the rest state solution. However, below the threshold, the solution was initiated with a weak convective flow. As displayed in Fig. 4, the solution was amplified above the threshold and decayed below. The time evolution of the flow intensity is displayed in Fig. 4. A curve fitting using exponential function was performed and the growth rate was computed. For $Le$=1, the two Rayleigh number values were 1100 and 1250 and the corresponding obtained growth rate was -0.7798 and 0.5640, respectively. The threshold for the onset of convection was obtained when p=0, so by interpolation, it was found that $Ra_{TC}^{sup} = 1187.04$. Redoing the calculations for various Lewis number, see Table 1, it was found that $Ra_{TC}^{sup}$ obeyed the following relationship with a great accuracy:

$$Ra_{TC}^{sup} = \frac{2374.08}{Le + 1}$$

The analytical expression of $Ra_{TC}^{sup}$ and the numerical results are depicted in Fig. 5 with a very good agreement. The threshold of subcritical convection existed only for the natural convection when $Le$≠1, and the values are tabulated in Table 1. The values were obtained by decreasing progressively the Rayleigh number using a small increment until a jump to conductive state occurred. Both the subcritical and supercritical values decreased with increasing Lewis number. At $Le$=1, subcritical convection was absent.

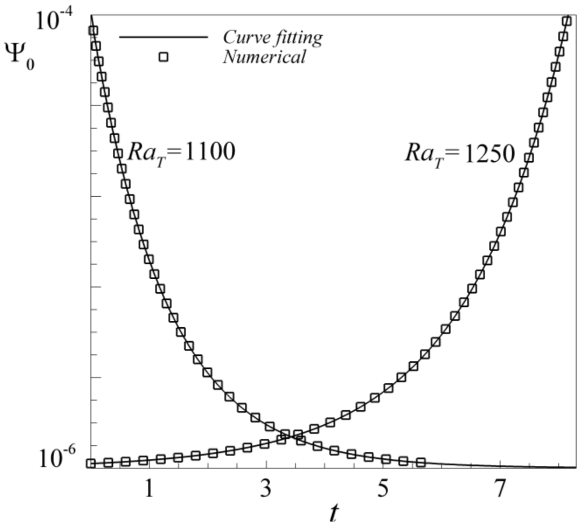


Figure 4: Flow intensity time histories below and above the threshold of supercritical convection for $Le$=1.

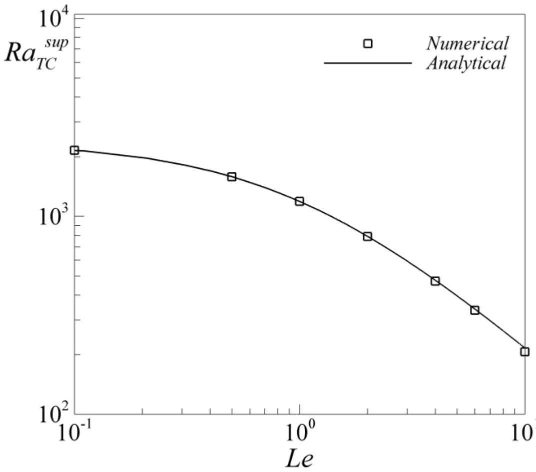


Figure 5: Supercritical Rayleigh number as function of the Lewis number.

Table 1: Computed subcritical and supercritical Rayleigh numbers as function of the Lewis number.

| $Le$ | 0.1 | 0.5 | 1 | 2 | 10 |
|---|---|---|---|---|---|
| $Ra_{TC}^{sup}$ | 2152.5 | 1581.5 | 1187.04 | 789.2 | 206.5 |
| $Ra_{TC}^{sub}$ | 1830 | 1480 | - | 740 | 185 |

Figure 6 displays the bifurcation diagram for $Le$=2. Below $Ra_{TC}^{sub}$, the system was unconditionally stable and the solution was characterized by a pure conductive state. Between the critical values, $Ra_{TC}^{sub}$and $Ra_{TC}^{sup}$, the convective could be triggered only by a finite amplitude perturbation. The conductive state remained stable to infinitesimal perturbation. Above $Ra_{TC}^{sup}$ , the conductive state became unconditionally instable. Any perturbation, regardless its amplitude, could trigger a convective state. Overstability convection was not observed in the present investigation.

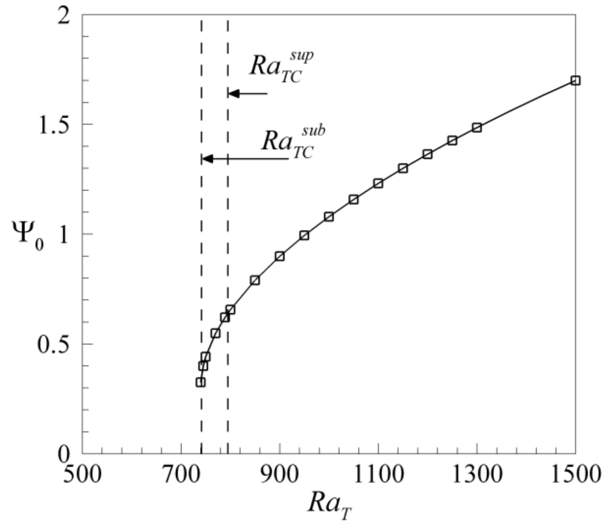


Figure 6: Bifurcation diagram in terms of the flow intensity as function of the Rayleigh number for *Le*=2.

## 5. Conclusion

A numerical study was performed on thermosolutal convection in a tilted square fluid enclosure subject to cross fluxes of heat and solute. The situation where the enclosure was tilted 45° and the buoyancy ratio is equal to unity (*N*=1) was considered. The co-existence of natural and anti-natural convective solutions was demonstrated. The thresholds for onset of supercritical and subcritical convection were obtained and depended on the Lewis number. Overall, the flow intensity and the heat and mass transfer rates increased with the Rayleigh increase. The effect of the increase of the Lewis number was to reduce the flow intensity and increase the mass transfer rate.

**Nomenclature**

$D$ mass diffusivity
$g$ gravity
$l$ dimension of the cavity
$Le$ Lewis number, $\alpha/D$
$Nu_m$ Nusselt number
$N$ buoyancy ratio, $\beta_S \Delta S^* / \beta_T \Delta T^*$
$Pr$ Prandtl number, $\nu/\alpha$
$Ra_T$ Thermal Rayleigh number, g $\beta_T \Delta T^* l^{*3} / \nu\alpha$
$S$ non-dimensional concentration
$Sh_m$ Sherwood number
$T$ dimensionless temperature
$t$ dimensionless time
$u$ dimensionless velocity in $x$ direction
$v$ dimensionless velocity in $y$ direction
$x, y$ dimensionless coordinates axis

Greek symbols
$\beta_S$ coefficient of volumetric expansion with concentration
$\beta_T$ coefficient of volumetric expansion with temperature
$\Delta S^*$ concentration difference
$\Delta T^*$ temperature difference
$\alpha$ thermal diffusivity
$\nu$ Kinematic viscosity
$\psi$ dimensionless stream function
$\omega$ dimensionless vorticity


**References**

[1] R.C. Paliwal, C.F. Chen "Double-diffusive instability in an inclined fluid layer. Part 1. Experimental investigation,", J. Fluid Mech, Vol. 98, part 4, pp. 755-768,(1980).

[2] R.C. Paliwal, C.F. Chen "Double-diffusive instability in an inclined fluid layer. Part 2. Stability analysis,",J. Fluid Mech, Vol. 98, part 4, pp. 769-785, (1980).

[3] G. De Vahl Davis "Natural convection of air a square cavity a bench mark numerical solution ,", international journal for numerical methods in fluids, Vol. 3, pp. 249-264, (1983).

[4] M. Napolitano, G. Pascazio, L. Quantapelle "A review of vorticity condition in the numerical solution of the $\zeta - \psi$ equation ,", Computer and fluids, Vol. 28, pp. 139-185, (1999).

[5] M. Mamou, P. Vasseur, M. Hasnaoui "On numerical stability analysis of double-diffusive convection in confined enclosures,", J. Fluid Mech, Vol. 433, pp. 209-250, (2001).