Three-dimensional numerical study of the effect of heating sources dimension on natural convection in a cavity submitted to constant heat flux

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Résumé : La convection naturelle dans une cavité cubique, discrètement chauffée, est étudiée numériquement en utilisant une formulation volumes finis tridimensionnelle. Deux portions chauffantes sont placées sur la paroi verticale gauche de la cavité, alors que le reste de la paroi considérée est adiabatique. La paroi verticale opposée est maintenue à une température uniforme froide et les autres parois sont adiabatiques. Les effets des dimensions des sections chauffantes ε ($0.15 \le \varepsilon \le 0.35$) et du nombre de Rayleigh Ra ($10^3 \le Ra \le 10^7$) sur l'écoulement du fluide et le transfert de chaleur à l'intérieur de la cavité sont étudiés. Des lignes de courant et des isothermes ainsi que les variations du nombre de Nusselt moyen sont présentés pour différents ensembles des paramètres gouvernants.

Keywords : Three dimensional natural convection; heated sections dimension; constant heat flux.

1. Introduction

The problem of electronic components cooling is often encountered in practical devices. In fact, in thermal control of electronic systems, a careful attention is necessary to ensure an optimal evacuation of the heat surplus. Natural convection represents a simple and low cost mode of cooling, especially for low gradients temperature. Besides this application, natural convection process is also encountered in many practical cases, like solar collectors, buildings design, radiators…. Hence, the problem of natural convective heat transfer in enclosures has been studied extensively. A comprehensive review of this topic is given by Bejan et al. [1] and Goldstein et al. [2] for different combinations of geometrical and thermal imposed conditions. However, in most of these works, the studied configurations are two dimensional cavities, partially heated, with one or more heating portions [3-4]. Few works has considered the three-dimensional natural convection [5] which gives a more realistic presentation of the fluid motion and the heat exchange within the cavity.

Hence, the purpose of the present investigation is to study numerically the fluid flow and heat transfer induced by two heat sources embedded on the left vertical wall of a cubical cavity and submitted to constant heat flux q''. The rest of the considered wall is adiabatic while the temperature of the opposite vertical wall is maintained at a uniform cold temperature Tc. The governing parameters are the Rayleigh number Ra ($10^3 \le Ra \le 10^7$) and the heating sections dimension $\varepsilon = D / H$ ($0.15 \le \varepsilon \le 0.35$). The Prandtl number and the aspect ratio $Ax = H / L$ are fixed respectively to 0.71 and 1.

2. Problem formulation

The schematic configuration of the considered three-dimensional cubic cavity, coordinates and boundary conditions are shown in figure 1. Two heat sources are integrated on the left vertical wall of the cavity and submitted to constant heat flux q''. The rest of the considered wall is adiabatic while the temperature of the opposite vertical wall is maintained at a uniform cold temperature Tc. The other walls are adiabatic. The considered fluid is incompressible, steady-state, newtonian and verifying the Boussinesq approximation.

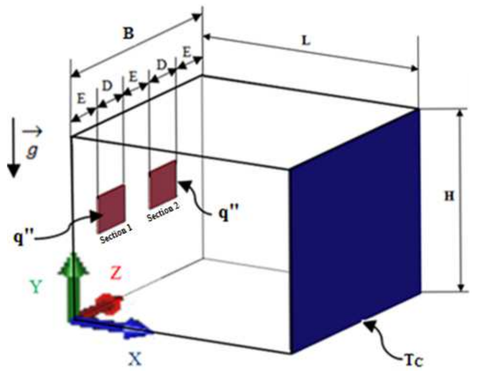

Figure 1 : Studied configuration and coordinates

The governing equations for laminar steady convection, using the Boussinesq approximation and neglecting the viscous dissipation, are expressed in the following dimensionless form:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}+\frac{\partial W}{\partial Z}=0 \tag{1}$$

$$\frac{\partial U}{\partial \tau}+\frac{\partial}{\partial X}(UU)+\frac{\partial}{\partial Y}(VU)+\frac{\partial}{\partial Z}(WU)=-\frac{\partial P}{\partial X}+\mathrm{Ra}\,\mathrm{Pr}\,\theta+\mathrm{Pr}(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}+\frac{\partial^2 U}{\partial Z^2}) \tag{2}$$

$$\frac{\partial V}{\partial \tau}+\frac{\partial}{\partial X}(UV)+\frac{\partial}{\partial Y}(VV)+\frac{\partial}{\partial Z}(WV)=-\frac{\partial P}{\partial Y}+\mathrm{Ra}\,\mathrm{Pr}\,\theta+\mathrm{Pr}(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}+\frac{\partial^2 V}{\partial Z^2}) \tag{3}$$

$$\frac{\partial W}{\partial \tau}+\frac{\partial}{\partial X}(UW)+\frac{\partial}{\partial Y}(VW)+\frac{\partial}{\partial Z}(WW)=-\frac{\partial P}{\partial Z}+\mathrm{Pr}(\frac{\partial^2 W}{\partial X^2}+\frac{\partial^2 W}{\partial Y^2}+\frac{\partial^2 W}{\partial Z^2}) \tag{4}$$

$$\frac{\partial \theta}{\partial \tau}+\frac{\partial}{\partial X}(U\theta)+\frac{\partial}{\partial Y}(V\theta)+\frac{\partial}{\partial Z}(W\theta)=(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}+\frac{\partial^2 \theta}{\partial Z^2}) \tag{5}$$

Where U, V and W are the velocity components in the X, Y and Z directions, respectively, P is the pressure, τ is the time and θ the temperature. The non-dimensional variables used in these equations are defined by:

$$(X,Y,Z)=(\frac{x}{H},\frac{y}{H},\frac{z}{H}),\ (U,V,W)=(\frac{uH}{\alpha},\frac{vH}{\alpha},\frac{wH}{\alpha}),\ P=\frac{H^2}{\alpha^2}p,\ \tau=\frac{\alpha}{H^2}t \text{ and } \theta=\frac{T-T_C}{q''H}k \tag{6}$$

Where α, υ and k represent respectively the thermal diffusivity, the kinematic viscosity and the thermal conductivity of the fluid.

In the above equations, the parameters Pr and Ra denote the Prandtl number, and the Rayleigh number, respectively. These parameters are defined as :

$$\mathrm{Pr}=\frac{\upsilon}{\alpha} \quad \text{and} \quad \mathrm{Ra}=\frac{g\,\beta\,q''H^4}{\alpha\,\upsilon\,k} \tag{7}$$

The local Nusselt number and the total average Nusselt number are respectively defined by:

$$\mathrm{Nu}(y,z)=\frac{q''H}{(T(x,y)\big|_{x=0}-T_c)k}=\frac{1}{\theta(Y,Z)\big|_{X=0}} \tag{8}$$

$$\mathrm{Nu}=2\iint \mathrm{Nu}(y,z)dydz \tag{9}$$

Where $\theta(Y,Z)$, in equation (8), is the local dimensionless temperature at a given point of the heat source surface.

3. Numerical method

The governing equations (Navier-Stokes and energy equations) are discretized by the finite volume method adopting the power low scheme. The Alternating Direction Implicit scheme (ADI) is then used for solving the obtained algebric system. The tri-diagonal system obtained in each direction is solved using the THOMAS algorithm. The accuracy of the numerical code was checked by comparing results obtained for constant heating temperature with those previously published by Frederick et al. [6] in the case of cubical enclosure with a partially heated wall and Fusegi et al. [7] in the case of cubical enclosure with a completely heated vertical wall. Finally, the non-uniform staggered grid of 41×41×41 nodes was estimated to be appropriate for the present study since it permits a good compromise between the computational cost (a significant reduction of the execution time) and the accuracy of the obtained results. The optimal time step was also found to be equal to 10^{-3} after multiple tests.

4. Results and discussions

The results presented in this section were obtained for Rayleigh numbers Ra ranging between 10^3 and 10^7 and the heating sections dimension ε between 0.15 and 0.35. The Prandtl number Pr and the aspect ratio $Ax = H / L$ are respectively fixed at 0.71 and 1.

4.1. Isotherms and streamlines

In order to visualize the flow and the temperature distribution within the studied area, streamlines and isotherms in 3D as well as isotherms on the heating sections are respectively shown in figures 2a and 2b, for $\boldsymbol{\varepsilon}$ = 0.35 and Ra = 10^6. It is seen that the fluid flow consists of a big and unique cell occupying the entire cavity. The fluid motion leads the heat from the active sections through the cavity. High values of the temperature are normally observed in the upper part of the enclosure. This trend is also encountered in the isotherms presented over the heating sections, as shown in figure 2c.

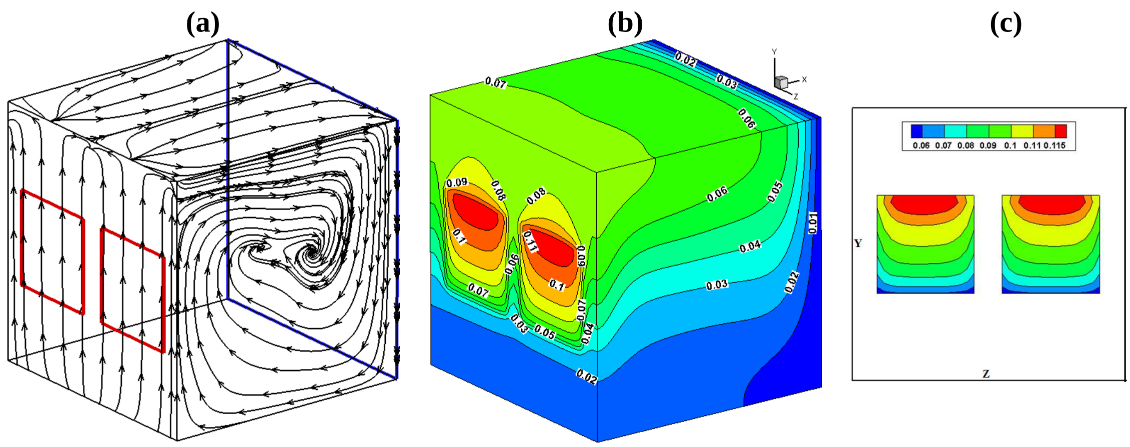

Figure 2 : Streamlines 3D (a), isotherms 3D (b) and isotherms on the sections (c) for Ra = 10^6 and $\boldsymbol{\varepsilon}$ = 0.35.

A presentation of isotherms and streamlines for different plans ($0 \le Z \le 1$) shows a good symmetry with respect to plane Z = 0.5, due to the adopted geometry and thermal boundary conditions. Hence, the plane Z = (2 - $\boldsymbol{\varepsilon}$) / 6, perpendicular to the middle of the section 1 and characterized by high thermal activity, is considered as a representative plan of the fluid motion and the heat transfer. In addition, streamlines and isotherms in this plan presents a perfect symmetry relative to the plane Z = 0.5 and are therefore identical to those obtained in the plane Z = (4 + $\boldsymbol{\varepsilon}$) / 6 perpendicular to the middle of the section 2.

Hence, in order to highlight the effect of the heating sections dimensions $\boldsymbol{\varepsilon}$ ($0.15 \le \boldsymbol{\varepsilon} \le 0.35$), the hydrodynamic and thermal fields in the cavity are shown in figure 3 for Ra = 10^7 and four dimensions $\boldsymbol{\varepsilon}$ = 0.15, $\boldsymbol{\varepsilon}$ = 0.20, $\boldsymbol{\varepsilon}$ = 0.25 and $\boldsymbol{\varepsilon}$ = 0.35. For the four cases, figures 3a and 3b present respectively the streamlines and the isotherms in the plane (2 - $\boldsymbol{\varepsilon}$) / 6 perpendicular to the middle of the section 1. Figures 3c present the isotherms over the two heating sections. For all the considered cases, the flow consists of a unique cell with cores intensity depending on the sections dimension. In addition, the temperatures at sections increase passing from the value θ_{max} = 0.0505 for $\boldsymbol{\varepsilon}$ = 0.15 to the value θ_{max} = 0.0733 for $\boldsymbol{\varepsilon}$ = 0.35. Their maximum values are reached at the midpoints of the upper edges of the sections

4.2. Nusselt number :

Figure 4 represent the total average Nusselt number, calculated at the two heater sections for Ra ranging between 10^3 and 10^7 and different values of $\boldsymbol{\varepsilon}$ ($0.15 \le \boldsymbol{\varepsilon} \le 0.35$). Note that the average Nusselt numbers calculated for each heating section are found to be identical for all the considered cases. As expected, the total average

Nusselt number increases with the Rayleigh number and especially from Ra = 10^4. Figure 4 also shows that for fixed Rayleigh number, the average Nusselt number decreases with increasing ε. For example, for Ra = 10^6 and ε = 0.15, the average Nusselt number is 9.75%, 17.42% and 29.38% higher than the values corresponding to ε = 0.20, ε = 0.25 and ε = 0.35 respectively. This trend is also encountered in previous works of [3-4].

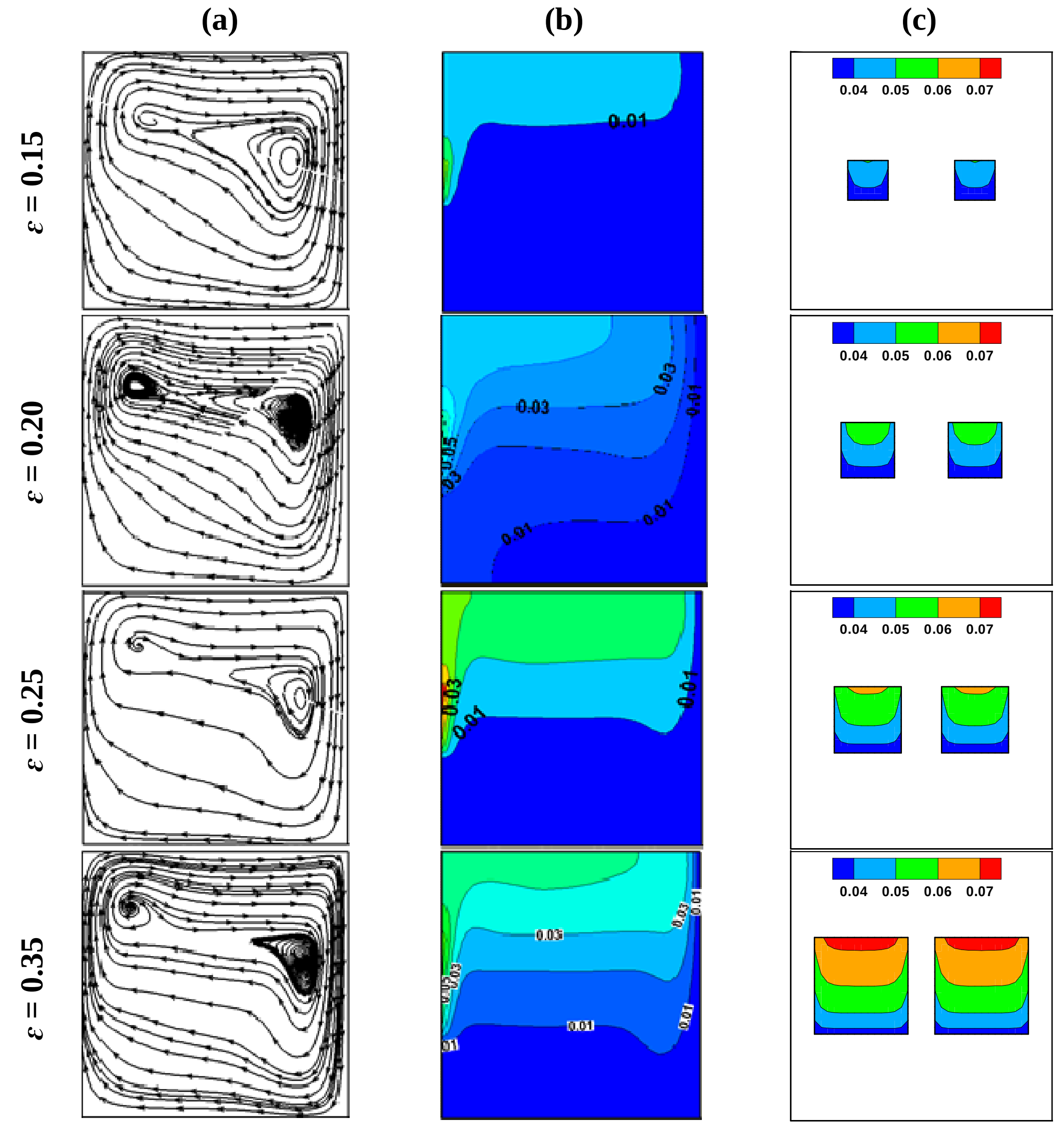

Figure 3 : Streamlines (a) and isotherms (b) in the plane Z = $(2 - \varepsilon) / 6$ and isotherms on the sections (c) for Ra = 10^7 and $0.15 \leq \varepsilon \leq 0.35$.

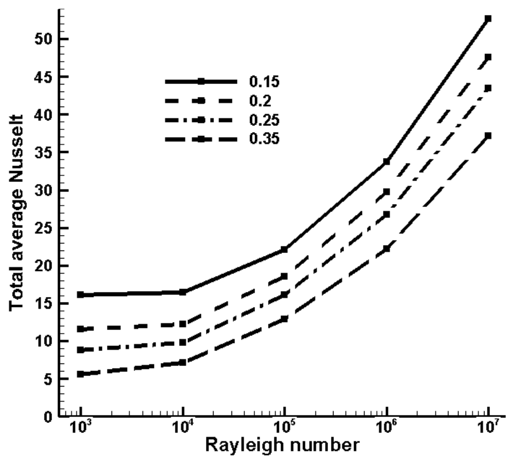

Figure 4 : Variation of the total average Nusselt number with Rayleigh number for $0.15 \leq \varepsilon \leq 0.35$.

Conclusion

Three dimensional naturel convection in a cavity discretely heated from the side has been studied for different sets the governing parameters (Rayleigh number and heating sections dimensions) and leads to the following conclusions:

- The fluid flow consists of a big cell occupying the entire cavity for all the considered cases ($10^3 \le Ra \le 10^7$) and ($0.15 \le \varepsilon \le 0.35$);
- The total average heat transfer, calculated in the two heating sections, increases with the Rayleigh number Ra and very significantly beyond $Ra = 10^4$;
- For fixed Rayleigh number, the average Nusselt number decreases with increasing ε.

Nomenclature

B Depth of the cavity, m
D Side of the square hot section, m
g Gravitational acceleration, m/s^2
H Height of the cavity, m
k Thermal conductivity, $w/(m.K)$
L Cavity length, m
Nu Total average Nusselt number
Pr Prandtl number, υ/α
p Non-dimensional pression
P Pression, N/m^2
q'' Heat flux, w/m^2
Ra Rayleigh number, $g\beta q'' H^4/(\upsilon\alpha k)$
u,v,w Dimensional velocities, m/s^2
U,V,W Dimensionless velocities
x,y,z Dimensional coordinates, m
X,Y,Z Dimensionless cartesian coordinates

Symboles grecs
A Thermal diffusivity, $m^2.s^{-1}$
θ Non-dimensional temperature
β Volumetric thermal expansion coefficient, K^{-1}
ε Non-dimensional side of the square hot section
μ Dynamique viscosity, $Kg/(m.s)$
υ Kinematic viscosity, m^2/s
ρ Density, Kg/m^3

Subscripts
max maximum value
c cold

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