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# Free convection over a vertical flat plate embedded in a saturated porous medium with variable heat source and radiation flux

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**Abstract:** The main object of this work is to study the effect of thermal radiation on the free convection flow induced by a heated vertical flat plate embedded in a saturated porous medium with an internal heat source. The temperature distribution of the plate has been assumed of the form $T_w(x) = T_\infty + A\,x^\lambda$, and the applied lateral mass flux is subjected to $x^{(\lambda-1)/2}$ quantity, where $x$ is the distance measured along the vertical plate and $\lambda$ is the constant temperature exponent. The non- linear equations of the similarity analysis with boundary layer conditions have been solved numerically using a fifth-order Runge-Kutta scheme coupled with the shooting method. Also, numerical results are obtained and displayed graphically to illustrate the effects of the thermal radiation parameter $N_R$, the temperature exponent $\lambda$ and the fluid suction/injection on the velocity and temperature fields.

**Key words** :

Free convection, saturated porous medium, suction/injection, internal heat source, thermal radiation.

## 1. Introduction

In recent years, free convection flow of fluids through porous medium has attracted the attention of a number of authors in many geophysical and engineering applications. Such applications are flow of chemical reactor engineering, soil physics, geohydrology, oil extraction, transport processes in aquifers and biological systems. Many studies have appeared concerning the interaction of radiative flux with thermal convection flows. In view of this, Hossain and Pop (1997, [1]) have analyzed the radiation effect on a free convection flow along an inclined plate in a porous space. Ghosh and Bég (2008, [2]) have analysed theoretically the radiative effects on transient free convection heat transfer past a hot vertical surface in porous media. Cortell (2010, [3]) have highlighted the effect of the internal heat generation and radiation on a certain free convection flow. Badruddin et al ( 2006, [4]) studied the free convection and radiation for a vertical wall with varying temperature embedded in a porous medium. Salleh et al (2010, [5]) have investigated numerically the free convection over a permeable horizontal flat plate embedded in a porous medium with radiation effects and mixed thermal boundary conditions.

In this work, we propose to study the effects of fluid suction/injection, temperature exponent and thermal radiation on free convection over a vertical flat plate embedded in a saturated porous medium with a variable internal heat source.

## 2. Mathematical model and similarity analysis

The problem is to study the convection around a heated vertical flat plate embedded in a saturated porous medium, in the presence of an internal heat source. The distribution of the plate temperature varies according to the relationship $T_w(x) = T_\infty + A\,x^\lambda$, where $T_\infty$ is the temperature away from the plate assumed constant, $A$ is a positive constant, $\lambda$ is the exponent of the temperature supposed constant. Cartesian coordinates $x$ and $y$ are measured, respectively, along and perpendicular to the plate. The coordinates system and the flow configuration are shown in Fig.1.

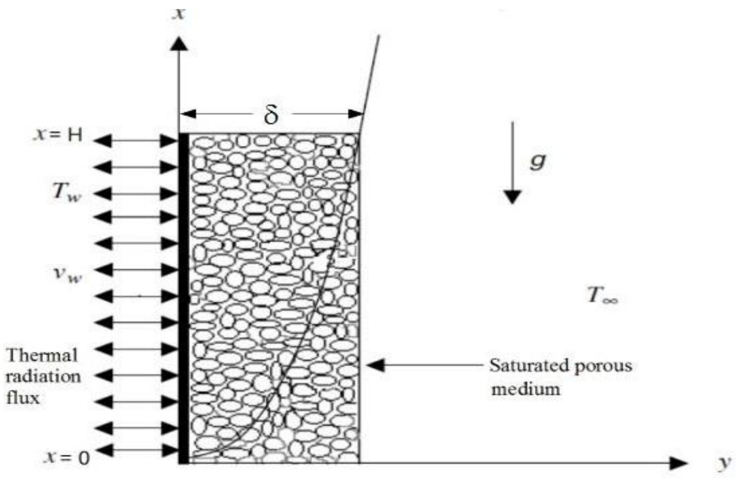


**Fig.1**: Vertical flat plate in a saturated porous medium.

Taking into account certain assumptions, the governing equations for the problem can be written as :

$$\begin{cases} \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 & (1) \\ u = \dfrac{g\beta K}{\nu}(T - T_\infty) & (2) \\ u\dfrac{\partial T}{\partial x} + v\dfrac{\partial T}{\partial y} = a\dfrac{\partial^2 T}{\partial y^2} + \dfrac{\varphi}{\rho C_p} - \dfrac{1}{\rho C_p}\dfrac{\partial Q_r}{\partial y} & (3) \end{cases}$$

The boundary conditions associated with the problem are:

$$\begin{cases} y=0 \quad x\geq 0 \quad v=V_w, \ T=T_w & (4) \\ y\to\infty \quad x\geq 0 \quad u=0, \quad T=T_\infty & (5) \end{cases}$$

where $u$ and $v$, are respectively, the velocity components along $x$ and y axes. $T$ is the temperature of fluid and $\varphi$ is the internal source of heat. The constants $\nu$, $k$, $a$, $g$ and $\rho$ are respectively, the kinematic viscosity, the permeability, the thermal diffusivity, the gravitational acceleration and the density. $C_p$ and $\beta$ are respectively the specific heat at constant pressure and the coefficient of thermal expansion, $V_w = B\ x^{(\lambda-1)/2}$ is the lateral mass flux, where $B$ is a constant. The radiation flux on the basis of the Rosseland diffusion model for radiation heat transfer is expressed as:

$$Q_r = -\left(\frac{4\sigma}{3\chi}\right)\frac{\partial T^4}{\partial y} \qquad (6)$$

Equations (1) to (3) are non-linear, simultaneous partial differential equations (PDEs) and to obtain solutions for them are extremely difficult. Consequently, we adopt the method of seeking similarity solutions in order to reduce the system of PDEs (1) to (3) into a set of ordinary differential equations (ODEs).

After substitution and development, the system of equations (1), (2) and (3) can be written as:

$$\begin{cases} (1+\frac{4}{3}N_R)f'''(\eta) - \lambda f'^2 + \frac{\lambda+1}{2}f(\eta)\ f''(\eta) + e^{-\eta} = 0 & (7) \\ \eta = 0 \quad f(0) = f_w \quad , \quad f'(0) = 1 & (8) \\ \eta\to\infty \quad f'(\infty) = 0 & (9) \end{cases}$$

with $\eta$ is the similarity variable and $f$ is the similarity function.

The ordinary differential equation (7), subject to the boundary conditions (8) and (9) is solved numerically by the fifth-order Runge-Kutta scheme, associated with the shooting iteration technique.

## 3. Results

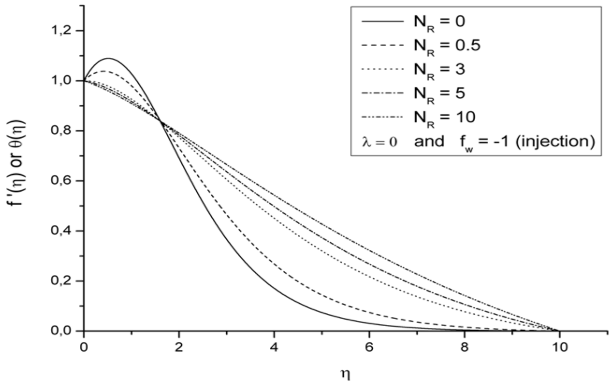


Fig. 2 : Velocity or temperature profiles at $\lambda = 0$ , $f_W = -1$ and for several values of $N_R$ .

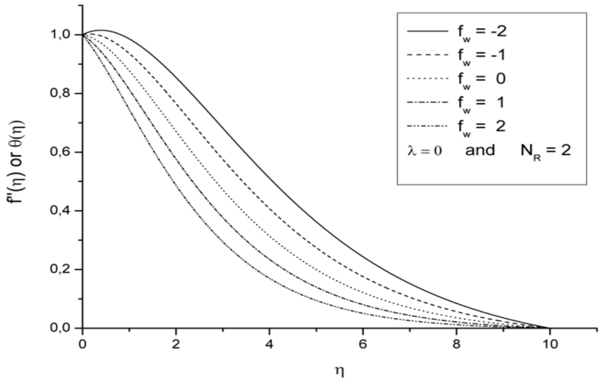


Fig. 3 : Velocity or temperature profiles at $\lambda = 0$ , $N_R = 2$ and for several values of $f_W$ .

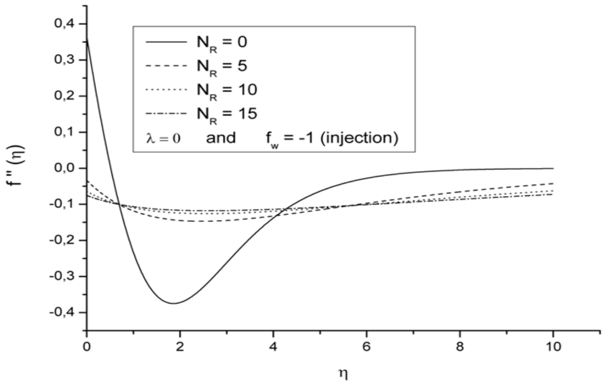


Fig.4 **:** Shear stress profiles for various values of $N_R$ , $\lambda = 0$ and for $f_W = -1$

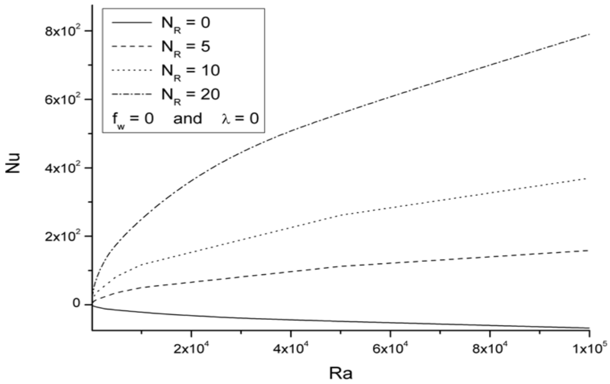

Fig. 5 : Influence of $Ra$ on $Nu$ at $\lambda = 0$ and $f_w = 0$ and for various values of $N_R$ .

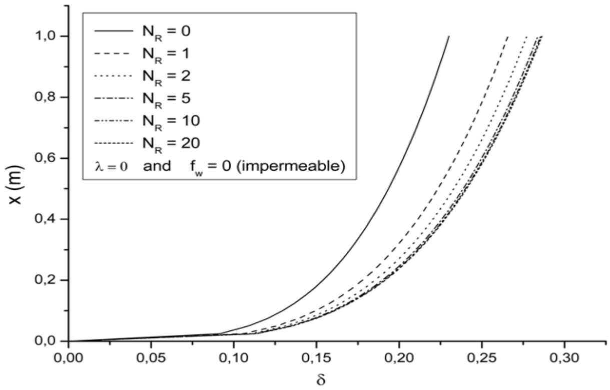

Fig.6 : Boundary layer thickness profile for $\lambda = 0$ , $f_W = 0$ and for various values of $N_R$ .

## 4. Discussion and conclusion

In this paper, we discuss the thermal radiation, the suction/injection parameter and the temperature exponent parameter effects on free convective flow over a heated vertical flat plate embedded in saturated porous medium with a variable internal heat source. The set of governing equations and the boundary conditions are reduced to ordinary differential equations with appropriate boundary conditions. Furthermore, the similarity equations are solved numerically by using the fifth-order Runge-Kutta scheme associated with the shooting method. The influence of thermal radiation parameter $N_R$, suction/injection parameter $f_W$ and temperature exponent parameter $\lambda$ on velocity or temperature, dimensionless shear stress, Nusselt number and boundary layer thickness profiles have been examined and discussed in detail. From the present numerical investigation we conclude that:

- The presence of the thermal radiation term in the energy equation increases the velocity or the temperature distribution.
- The reduction in velocity or temperature profile at any point of the flow field is faster as the suction parameter becomes larger.
- The minimum shear stress increases with $N_R$ increase and this minimum moves to the plate surface for more suction.
- The shear stress at the plate surface decreases with the presence of thermal radiation when compared with the case where the thermal radiation is absent for the values of the temperature exponent parameter $\lambda = 0$ , $\lambda = 1/3$ and $\lambda = 1$ and it also decreases as the suction increase.
- The boundary layer thickness increases with the increase of the thermal radiation parameter and it is clear that the larger values of this parameter has no significant influence on the boundary layer thickness.
- The effect of Rayleigh number which characterizes the intensity of natural convection was highlighted. The heat transfer increases with the Rayleigh number, and this increase and even more important than increasing $N_R$.

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