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Time periodic natural convection coupled to thermal radiation in a square cavity subjected to cross temperature gradients

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Abstract : Periodic coupled natural convection and surface radiation within a square cavity, filled with air and submitted to discrete heating and cooling from all its walls, is studied numerically. The thermally active elements are centrally located on the walls of the cavity. The parameters governing the problem are the amplitude and the period of the temporally sinusoidal temperature, the emissivity of the walls, the relative lengths of the active elements and the Rayleigh number. The effect of such parameters on flow and thermal fields and the resulting heat transfer is examined. It is shown that the flow structure can present complex behavior, depending on the emissivity and the amplitude and period of the exciting temperature. The rate of heat transfer is generally enhanced in the case of sinusoidal heating. Also, the resonance phenomenon existence, characterized by maximum fluctuations in flow intensity and heat transfer, is proved in this study.

Keywords: Natural convection, thermal radiation, heatlines, cross gradients of temperature, periodic heating, resonant heat transfer, numerical study.

1. Introduction

Combined natural convection and surface radiation in closed cavities have been extensively studied using numerical simulations and experiments owing to the practical importance of such a configuration in many engineering applications (convective heat losses from solar collectors, thermal design of buildings, air conditioning and recently, the cooling of electronic components). The majority of the existing studies, which are of numerical nature, concerned with rectangular cavities where the temperature gradient is either horizontal or vertical, including different kinds of boundary conditions. Actually, much more complex boundary conditions may be encountered in practical cases where horizontal and vertical temperature gradients are simultaneously imposed across the cavity [1-4]. In these studies, the thermal boundary conditions were assumed to be either steady isothermal or constant heat flux wall conditions. However, in many engineering applications, the energy provided to the system is variable in time and gives rise to unsteady natural-convection flow. The power supply of electronic circuits by an alternating current, the collectors of solar energy, rooms housing and building hollow blocks, in which recirculation is periodically driven by daily solar heating, are concrete examples. This justifies the presence of some works in the literature in which the variable aspect of the thermal boundary conditions was considered [5-9]. Results of these studies showed that the buoyancy-induced flow resonates to a certain frequency of the periodic heat input and the resonance phenomenon is characterized by maximum fluctuations observed in the heat transfer evolution with the period of the time-dependent thermal excitation. In our knowledge, works dealing with time periodic combined natural convection-radiation in rectangular cavities subjected to crossed thermal gradients are non-existent. This work is, therefore, a contribution to the numerical study of the effect of periodic heating on natural convection and surface radiation within a square cavity filled with air and discretely heated and cooled from the four walls: two heating modes, called ***SB*** and ***SV***, are considered. They correspond to bottom and vertical left elements sinusoidally heated in time, respectively. The effect of control parameters on heat transfer and fluid flow within the cavity is examined.

2. Problem formulation

The configurations under study, together with the system of coordinates, are depicted in Fig. 1. The 2D flow is conceived to be laminar and incompressible with negligible viscous dissipation. All the thermophysical properties of the fluid are assumed constant except the density in the buoyancy term which is assumed to vary linearly with temperature (Boussinesq approximation); such a variation gives rise to the buoyancy forces. Taking into account the above-mentioned assumptions, the non-dimensional governing equations, written in vorticity-stream function and temperature (Ω,Ψ, T) formulation, are as follows:

$$\frac{\partial\Omega}{\partial t}+\frac{\partial(u\Omega)}{\partial x}+\frac{\partial(v\Omega)}{\partial y}=Ra\,Pr\frac{\partial T}{\partial x}+Pr\left(\frac{\partial^2\Omega}{\partial x^2}+\frac{\partial^2\Omega}{\partial y^2}\right) \quad (1)$$

$$\frac{\partial T}{\partial t}+\frac{\partial(uT)}{\partial x}+\frac{\partial(vT)}{\partial y}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2} \quad (2)$$

$$\frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}=-\Omega \quad (3)$$

$$u=\frac{\partial\Psi}{\partial y}\ ,\ \ v=-\frac{\partial\Psi}{\partial x}\quad \text{and}\quad \Omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y} \quad (4)$$

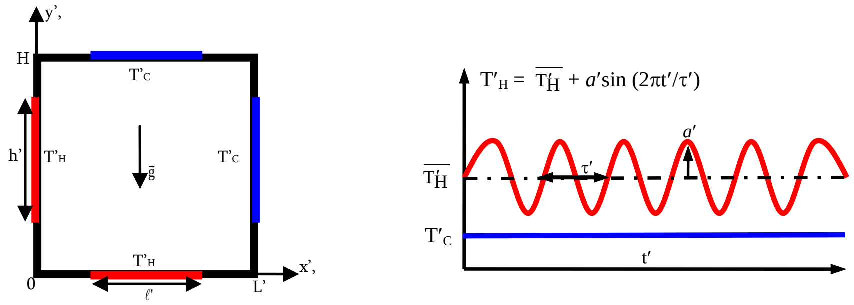


Fig. 1: Geometry of the problem with the imposed thermal excitations.

The dimensionless boundary conditions, associated to the problem are such:

$u = v = \Psi = 0$ on the cavity walls (5a)

$T = 0$ on the cooled elements (5b)

SB Mode $\begin{cases} T=1+a\sin(2\pi t/\tau) & \text{on the bottom heated element} \\ T=1 & \text{on the vertical heated element} \end{cases}$ (5c)

SV Mode $\begin{cases} T=1 & \text{on the bottom heated element} \\ T=1+a\sin(2\pi t/\tau) & \text{on the vertical heated element} \end{cases}$ (5cbis)

$-\frac{\partial T}{\partial n}+N_r Q_r=0$ on the adiabatic elements (5d)

The non-dimensional radiosity equation

$$J_i=\varepsilon_i\left(\frac{T_i}{T_r}+1\right)^4+(1-\varepsilon_i)\sum_{S_j}F_{ij}J_j \quad (6)$$

The non-dimensional net radiative heat flux leaving a surface S_i is evaluated by:

$$Q_r=\varepsilon_i\left[\left(\frac{T_i}{T_r}+1\right)^4-\sum_{S_j}F_{ij}J_j\right] \quad (7)$$

At each time step, the mean Nusselt numbers, characterizing the contributions of natural convection and thermal radiation through the heated walls, are evaluated as:

- on the vertical heated wall

$$Nu_{V(cv)}(t) = -\int_{0.5-\frac{B}{2}}^{0.5+\frac{B}{2}} \left(\frac{\partial T}{\partial x}\right)\bigg|_{x=0} dy \ ; \quad Nu_{V(rd)}(t) = \int_{0.5-\frac{B}{2}}^{0.5+\frac{B}{2}} \left(N_r Q_r\right)\big|_{x=0} dy$$

- on the horizontal heated wall

$$Nu_{H(cv)}(t) = -\int_{0.5-\frac{B}{2}}^{0.5+\frac{B}{2}} \left(\frac{\partial T}{\partial y}\right)\bigg|_{y=0} dx \ ; \quad Nu_{H(rd)}(t) = \int_{0.5-\frac{B}{2}}^{0.5+\frac{B}{2}} \left(N_r Q_r\right)\big|_{y=0} dx$$

The instantaneous convective, radiative and total Nusselt numbers across the whole cavity are defined respectively as:

$$Nu_{cv}(t) = Nu_{V(cv)}(t) + Nu_{H(cv)}(t) \qquad Nu_{rd}(t) = Nu_{V(rd)}(t) + Nu_{H(rd)}(t) \qquad Nu(t) = Nu_{cv}(t) + Nu_{rd}(t)$$

The mean Nusselt numbers, averaged in time over periods are calculated as:

$$\overline{Nu}_{cv} = \frac{1}{\tau_{cv}} \int_0^{\tau_{cv}} Nu_{cv}(t)dt \ , \quad \overline{Nu}_{rd} = \frac{1}{\tau_{rd}} \int_0^{\tau_{rd}} Nu_{rd}(t)dt \quad \text{and} \quad \overline{Nu} = \overline{Nu}_{cv} + \overline{Nu}_{rd}$$

where τ_{cv} and τ_{rd} are respectively the periods of the temporal variations of convective and radiative Nusselt numbers (they are identical in general).

The Heatlines (lines of constant heat function H) are defined through the first derivatives of the function H as follows [10]:

$$\frac{\partial H}{\partial y} = u\,T - \frac{\partial T}{\partial x}, \quad -\frac{\partial H}{\partial x} = v\,T - \frac{\partial T}{\partial y} \tag{8}$$

The dimensionless heat function equation can be derived easily from Eq. (8) as:

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = -\frac{\partial (vT)}{\partial x} + \frac{\partial (uT)}{\partial y} \tag{9}$$

The solution of Eq. (9) yields the values of the heat function in the nodes of the computational domain. The contour plots of the heat function values provide heatline patterns.

3. Method of solution

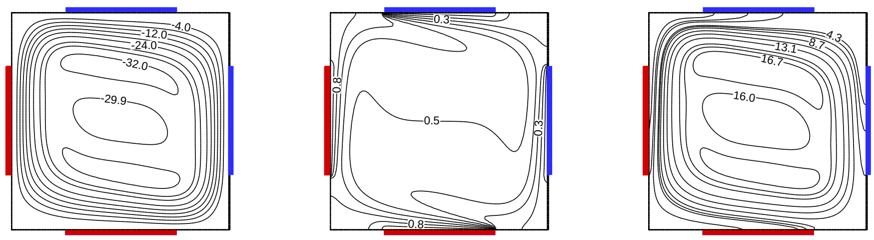
The non linear partial differential governing equations, Eqs. (1)-(3), were discretized using a finite difference technique. The first and second derivatives were approached by central differences. The integration of equations (1) and (2) was ensured by the Alternating Direction Implicit method (ADI). At each time step, the Poisson equation, Eq. (3), was treated by using the Point Successive Over-Relaxation method (PSOR). The set of Eqs. (6), representing the radiative heat transfer between the different elementary surfaces of the cavity, was solved by using the Gauss-Seidel method. The accuracy of the numerical model was checked by comparing results from the present investigation against those previously published by Akiyama and Chong [11] in the case of a differentially heated square cavity.

4. Results and discussion

The main parameters governing the problem are the amplitude of the exciting temperature ($0 \leq a \leq 1$), its period ($0.001 \leq \tau \leq 1$), the emissivity of the walls ($0 \leq \varepsilon \leq 1$), the Prandtl number, Pr, the Rayleigh number, Ra, and the relative length of active elements, B. To highlight the influence of a, τ and ε, the values of B, Pr and Ra are fixed to 0.5, 0.72 (air) and 10^6 respectively.

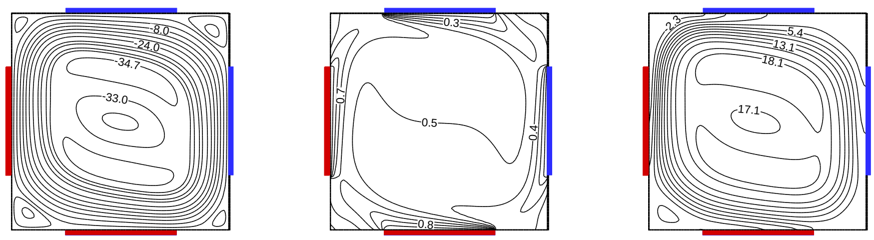
4.1. Streamlines, isotherms and heatlines in stationary flow (a = 0)

The effect of radiation on dynamical structure, temperature distribution and heat flux transport inside the cavity is illustrated in Fig. 2 (case of $\varepsilon = 0$) and Fig. 3 (case of $\varepsilon = 1$) for Ra = 10^6. In the absence of walls' radiation (ε = 0), the streamlines reveal the existence of a main clockwise square peripheral unicellular flow, slightly flattened at the corners, almost in contact with all the cavity's surfaces and surrounding three small cells in the central part of the cavity. The corresponding isotherms are tightened at the vicinity of the active walls indicating a good thermal interaction between the fluid and the active elements. The isotherms are however more spaced in the vicinity of the heated and cooled vertical walls compared to those near the horizontal active elements, indicating an advantage in favor of the latter in contributing to the convective heat exchange. An increase of ε up to 1 (Fig. 3) leads to more complicated and quite different flow structures. The contribution of radiation supports the three inner cells by increasing their size and intensity and favors the formation of two other vortices, of less importance, at the remaining corners of the cavity. A similitude in the shapes of heatlines and streamlines is observed indicating a predominant contribution of convection to the overall heat transfer.



(a) (b) (c)

Fig. 2: (a) Streamlines, (b) isotherms et (c) heatlines in the case of constant heating (a = 0) for ε = 0.



(a) (b) (c)

Fig. 3: (a) Streamlines, (b) isotherms et (c) heatlines in the case of constant heating (a = 0) for ε = 1.

4.2. Effect of the period τ on $\overline{\mathrm{Nu}}$

In the absence of radiation (ε = 0), variations of $\overline{\mathrm{Nu}}$ are presented in Fig. 4. It can be seen that $\overline{\mathrm{Nu}}$ increases with τ to a peak, characterizing a resonance phenomenon, for a critical value of τ, which is of 0.008 for the ***SB*** mode and 0.00825 for the ***SV*** one. By increasing the amplitude a of the variable temperature, the peak becomes more important and the resonance phenomenon becomes more pronounced, but without changing the critical periods.

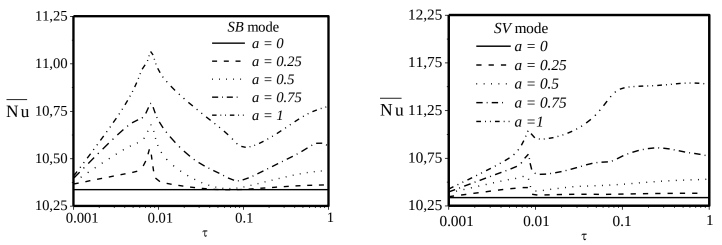


Fig. 4: Variations of $\overline{\mathrm{Nu}}$ with the period τ for ε = 0 and different values of a.

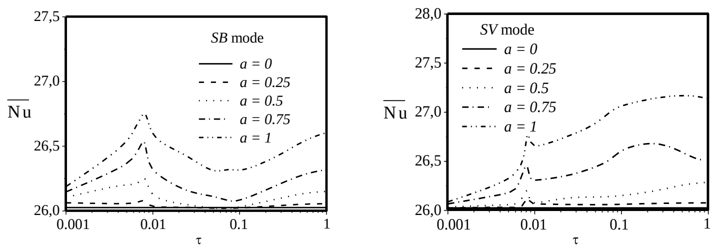


Fig. 5: Variations of $\overline{\mathrm{Nu}}$ with the period τ for ε = 1 and different values of a.

Also, it is important to note that beyond the critical value of τ, $\overline{Nu}$ decreases to a minimum value reached for τ_0 which depends on the imposed heating mode. In the case of ***SB*** mode, $\tau_0 = 0.09$, while in the case of the ***SV*** one, τ_0 is substantially less and is 0.01, values beyond which $\overline{Nu}$ increases until an asymptotic trend, depending on *a*. In the case of highly emissive walls ($\varepsilon = 1$), Fig. 5 shows that the total heat transfer are more enhanced in comparison with the case of $\varepsilon = 0$. When ε passes from 0 to 1, the enhancement of the time averaged heat transfer, $\overline{Nu}$, is of about 142 % at the resonance. The resonance phenomenon is obtained for the same critical periods for both considered heating modes.

Conclusion

The problem of periodic natural convection coupled with thermal radiation inside a square cavity, submitted to cross gradients of temperature, has been studied numerically. A resonance phenomenon, characterized by maximum fluctuations in flow intensity and heat transfer, is observed. The resonance period has the particularity of being independent vis-à-vis the excitation amplitude and the emissivity of the walls. With the exception of low values of the period of the exciting temperature, time variable heating, generally improves heat transfer compared with the case of a constant heating.

Nomenclature

F_{ij} view factor from S_i surface to S_j one

g acceleration due to gravity, m/s^2

J_i dimensionless radiosity, $J'_i/\sigma T'^4_C$

Nu average Nusselt number

Pr Prandtl number, $Pr = \nu/\alpha$

Ra Rayleigh number, $Ra = g\beta(T'_H - T'_C)H'^3/\alpha\nu$

T dimensionless fluid temperature

T_r dimensionless reference temperature, $T_r = T'_C/(T'_H - T'_C)$

t dimensionless time, $t = t'\alpha/H'^2$

(u,v) dimensionless horizontal and vertical velocities

(x,y) dimensionless coordinates, $(x, y) = (x', y')/H'$

Greek symbols

α thermal diffusivity of fluid, m^2/s

β thermal expansion coefficient of fluid, 1/K

λ thermal conductivity of fluid, W /(K $\times$ m)

ν kinematic viscosity of fluid, m^2/s

Ω dimensionless vorticity, $\Omega = \Omega' H'^2/\alpha$

Ψ dimensionless stream function, $\Psi = \Psi'/\alpha$

σ Stéfan-Boltzman constant

Subscripts, Superscripts

C cooled surface

H heated surface

$'$ dimensional variable

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