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Lattice-Boltzman simulations of free convection in an inclined square cavity partially heated and cooled from the sides and filled with nanofluids using heatline method

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Abstract
The Lattice-Boltzman method (LBM), is used to simulate natural convection in a square inclined cavity filled with CuO-water nanofluid. The cavity is submitted to partial and centered heating and cooling from two opposite sides with constant temperatures. The numerical code was successfully validated against available works using conventional numerical methods. The obtained results are analyzed for wide ranges of the governing parameters (Rayleigh number, Ra, volume fraction of the nanoparticles, φ and inclination of the cavity, δ). The obtained results are presented in terms of streamlines, isotherms, heatlines, average Nusselt numbers and also in terms of velocity and temperature profiles for various combinations of the governing parameters.

Key words: Lattice-Boltzman method, Heatlines concept, Nanofluid

1. Introduction

Natural convection heat transfer in enclosures is an important phenomenon in engineering systems due to its wide applications. Therefore, improvement in the heat transfer of these systems is an essential topic from an energy saving perspective since cooling process remains one of the most technical challenges in many industrial applications (including microelectronics and manufacturing). Various techniques have been tested in the past with the objective to enhance heat transfer rates in confined geometries with small volumes. The most innovative and most promising technique is the use of nanoscale particles suspended in the base liquid. This technique has generated considerable interest due to the potential of nanoparticules (Al_2O_3, CuO, Cu, Ag, TiO_2 …) to enhance the heat transfer rate in engineering systems.

Through the decades, researchers have analyzed the effect of discrete boundary conditions on natural convection in rectangular enclosures by using various numerical methods. For instance, Hasnaoui et al. [1] have used a finite difference technique to study natural convection in a rectangular cavity partially heated from below. The effect of the heating source position was analyzed and the multiplicity of solutions was demonstrated in the case where the heating element was centrally located. More recently, the finite volume [2] and the finite element [3] methods were used to study natural convection flows generated by partial heating and cooling in rectangular cavities. The problem of discrete thermal boundary conditions was extended to study natural convection problems in cavities filled by different nanofluids (CuO-water nanofluid [4], Cu-water nanofluid [5] …).

In the present study, the Lattice-Boltzman method (LBM), is used to simulate natural convection in a square cavity inclined with respect to the horizontal. The cavity is filled with the CuO-water nanofluid and submitted to partial and centered heating and cooling from two opposite side walls with constant temperatures. The D2Q9 Lattice model was used for both density and internal energy distribution functions. Due the space limitation, typical results in terms of streamlines, isotherms, heatlines, average Nusselt numbers and velocity and temperature profiles are illustrated for various combinations of the governing parameters.

2. Mathematical formulation

2.1. Position of the problem

The studied configuration is sketched in Fig. 1. It consists of a square cavity of length L with two partially active walls, centrally located on two opposite walls, and maintained at constant temperatures T_H and T_C ($T_H > T_C$), respectively. The length of the active elements is $L/2$ and the remaining surfaces are assumed adiabatic. The cavity is inclined at an angle δ with respect to the horizontal and filled with a CuO-water nanofluid. The flow is assumed Newotonien, laminar and incompressible which requires that the Mach number should not exceed 0.3. The nanofluid density is approximated by the Boussinesq standard model.

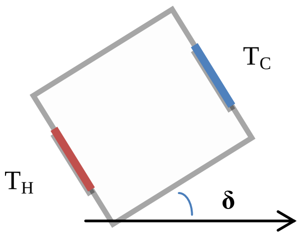


Figure 1: Studied configuration.

2.2. Overview of the Lattice Boltzmann method (LBM)

Historically, the Boltzmann equations, Eqs. (1)-(2), are written in the BGK approximation. In the LBM, the domain is decomposed into lattices of arrangements D2Q9 [6] and the double distribution function (DDF) is used in this study. Two functions are introduced in the model; the first one, $f_i(r,t)$, corresponds to velocity while the second one, $g_i(r,t)$, corresponds to temperature.

The lattice Boltzmann equation with an external force F can be written respectively for the fluid flow and temperature distribution as follows:

$$f_i(r + c_i\Delta t, t + \Delta t) = f_i(r,t) - \omega_v.\left(f_i(r,t) - feq_i(r,t)\right) + F_i\Delta t \tag{1}$$

$$g_i(r + c_i\Delta t, t + \Delta t) = g_i(r,t) - \omega_t.\left(g_i(r,t) - geq_i(r,t)\right) \tag{2}$$

The local equilibrium distribution functions, denoted $feq_i(r,t)$ and $geq_i(r,t)$ are expressed as:

$$feq_i(r,t) = \omega_i\rho\left[1 + \frac{\boldsymbol{c_i}.\boldsymbol{u}}{c_s^2} + \frac{1}{2}\frac{(\boldsymbol{c_i}.\boldsymbol{u})^2}{c_s^4} - \frac{1}{2}\frac{\boldsymbol{u}.\boldsymbol{u}}{c_s^2}\right] \tag{3}$$

$$geq_i(r,t) = \omega_i T\left[1 + \frac{\boldsymbol{c_i}.\boldsymbol{u}}{c_s^2}\right] \tag{4}$$

For $D2Q9$ arrangements, the discrete velocities $\boldsymbol{c_i}$ are defined as follows:

$$\boldsymbol{c_i} = c\begin{cases} 0 & for \quad i = 0 \\ \left(cos\left[(i-1)\frac{\pi}{2}\right], sin\left[(i-1)\frac{\pi}{2}\right]\right) & for\ \ i = 1,2,3,4 \\ \sqrt{2}\left(cos\left[(i-5)\frac{\pi}{2} + \frac{\pi}{4}\right], sin\left[(i-5)\frac{\pi}{2} + \frac{\pi}{4}\right]\right) & for\ \ i = 5,6,7,8 \end{cases} \tag{5}$$

And the weighting factors are given by:

$$\omega_0 = 4/9, \omega_1 = \omega_2 = \omega_3 = \omega_4 = 1/9 \text{ et } \omega_5 = \omega_6 = \omega_7 = \omega_8 = 1/36. \tag{6}$$

Then, the discrete external force is given by the following expression:

$$F_i = 3\omega_i F = 3\omega_i\rho g\beta(T - T_m)(cy_i.cos\delta + cx_i.sin\delta) \tag{7}$$

Where $T_m = (T_H + T_C)/2$

Finally, the macroscopic quantities can be derived from the following formulas:

Density $\rho(r,t) = \sum_{i=0}^{i=8} f_i(r,t)$ (9)

Momentum $\rho\boldsymbol{u}(r,t) = \sum_{i=0}^{i=8} \boldsymbol{c}_i f_i(r,t)$ (10)

Temperature $T(r,t) = \sum_{i=0}^{i=8} g_i(r,t)$ (11)

The Chapman-Enskog procedure allows the deduction of the incompressible Navier-Stokes equations from the LBM as follows:

$$\nabla.u = 0 \tag{12}$$

$$\frac{\partial u}{\partial t} + u.\nabla u = -\nabla P + \left(\frac{2\tau_v - 1}{6}\right)\nabla^2 u \tag{13}$$

$$\frac{\partial T}{\partial t} + \nabla.(uT) = \left(\frac{2\tau_T - 1}{6}\right)\nabla^2 T \tag{14}$$

By analogy, the kinematic viscosity υ and the thermal diffusivity α are then linked to the relaxation times by the following expression:

$$\upsilon = (\tau_v - 0.5)c_s^2\Delta t \qquad \alpha = (\tau_T - 0.5)c_s^2\Delta t \tag{15}$$

With τ_v, τ_T, c_s and Δt stay respectively for hydrodynamic relaxation parameter, thermal relaxation parameter, sound speed and time increment.

2.3. Heat function and Nusselt number

The heat function is introduced as a solution of the following equation:

$$\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}=\frac{(\rho Cp)_{nf}}{(\rho Cp)_f}\left[\frac{\partial}{\partial y}(uT)-\frac{\partial}{\partial x}(vT)\right] \tag{23}$$

The Point Successive Over Relaxation (PSOR) method was used to solve Eq. (23) with an optimum relaxation factor given by the Frankel formula for the lattices used.

The local Nusselt number, Nu, and the average Nusselt number, $\overline{\mathrm{Nu}}$, evaluated at the heated surface are respectively calculated as:

$$Nu_y=-\frac{k_{nf}}{k_f}\left(\frac{\partial T}{\partial x}\right)_{x=0} \tag{24}$$

$$\overline{\mathrm{Nu}}=-2k_{eff}\int_{0:25}^{0.75}\left(\frac{\partial T}{\partial x}\right)_{x=0}dy \tag{25}$$

2.4. Nanofluids properties

The effective density of the nanofluid is given by the expression:

$$\rho_{nf}=(1-\varphi)\rho_{bf}+\varphi\rho_{np} \tag{26}$$

Whereas its heat capacitance and a part of the Boussinesq term are:

$$(\rho Cp)_{nf}=(1-\varphi)(\rho Cp)_{bf}+\varphi(\rho Cp)_{np} \tag{27}$$

$$(\rho\beta)_{nf}=(1-\varphi)(\rho\beta)_{bf}+\varphi(\rho\beta)_{np} \tag{28}$$

With φ being the volume fraction of the solid particles and the subscripts *f*, *nf* and *np* stand for base fluid, nanofluid and solid, respectively. The viscosity of the nanofluid is expressed as follows:

$$\mu_{nf}=\frac{\mu_{bf}}{(1-\varphi)^{2.5}} \tag{29}$$

The effective thermal conductivity of the nanofluid can be approximated by the Maxwell–Garnetts model as:

$$k_{nf}=\left(\frac{k_{np}+2k_{bf}-2\varphi(k_{bf}-k_{np})}{k_{np}+2k_{bf}+\varphi(k_{bf}-k_{np})}\right)k_{bf} \tag{30}$$

2.4 Validation of the numerical code

The numerical code was succeffuly validated in terms of heatlines against the papers by Basak and Roy [7] (results not presented) and also in terms average Nusselt number in a square cavity filled with Cu-water nanofluid and provided with partially active side walls [5]. Comparative results, presented in Table 1, show very satisfactory agreement since the maximum difference remains within 3%.

Table 1

Validation in terms of average Nusselt number in the case of a square cavity filled with Cu-water nanofluid.

Ra →	**10^3**		**10^4**		**10^5**	
φ ↓	Ref. [5]	Our code	Ref. [5]	Our code	Ref. [5]	Our code
0	1.02	1.05 (2.9%)	2.01	1.96 (2.4%)	3.98	4.01 (3%)
0.1	1.24	1.23 (0.8%)	2.2	2.2 (0.%)	4.42	4.47 (1.1%)
0.2	1.6	1.56 (2.5%)	2.36	2.35 (0.4%)	4.84	4.81 (0.6%)

3. Results and discussion

3.1 Qualitative effects of δ and φ

In this section, the flow structure, the isotherms and the heatlines are illustrated numerically in Figs. 2(a-c'), using the LBM, for $Ra=10^5$ and three inclinations corresponding to different directions of the gravity with

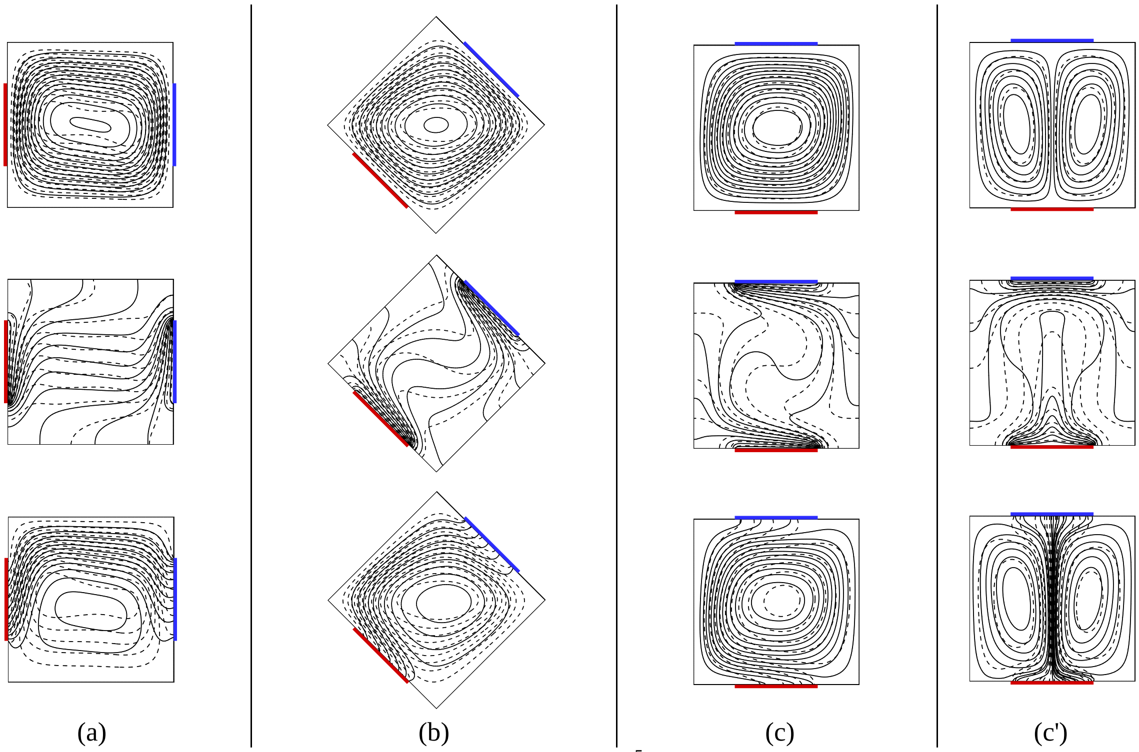


Figure 2: Streamlines, isotherms and heatlines for Ra = 10^5 and δ = 0° (a), 45° (b) and 90° (c and c').

respect to the active elements. These figures show qualitative comparisons between CuO-water nanofluid (dashed lines with $\varphi = 0.2$) and pure fluid (solid lines with $\varphi = 0$) behaviors in terms of streamlines (on the top), isotherms (on the center) and heatlines (on the bottom). It can be observed that both δ and φ have visible effects characterized by the demarcation between the dashed and solid lines. If we except the case $\delta = 90°$, case corresponding to heating from below for which two solutions are obtained (bicellular flow and monocellular flow turning either in the clockwise or counter-clockwise directions), the remaining inclinations generate unicellular clockwise flows. For the given value of Ra, the combined effects of the nanoparticles fractions and the inclination angle of the cavity have an apparent effect on the shapes of the structures and the thermally inactive area in the heatlines characterized by the closed lines. The latter, expanded or narrowed, have a direct effect on the width of the open lines (on the corridor ensuring heat transfer between the active elements), leading consequently to an enhancement or a reduction of heat transfer. In the isotherms we can observe that the isotherms are tightened in the vicinity of the heating and the cooling elements; there where the open lines of heatlines depart from the heating element and arrive at the cooling element. The central part of the cavity is the less active thermally due to the sparse isotherms and to the formation of closed heatlines in this region.

3.2 Effect of the cavity inclination

The effect of the inclination angle is illustrated in Fig. 3 in terms of average Nusselt number variations versus δ for $Ra = 10^5$ in the case of pure fluid (φ = 0) and nanofluid (φ = 0.2). For both values of φ, the variations observed are characterized by an increase towards a maximum (observed around δ = 22,33°/(37,43°) for φ = 0/(0.2)) followed by a monotonous decrease in the remaining range of δ. Note that, the case of δ = 90°

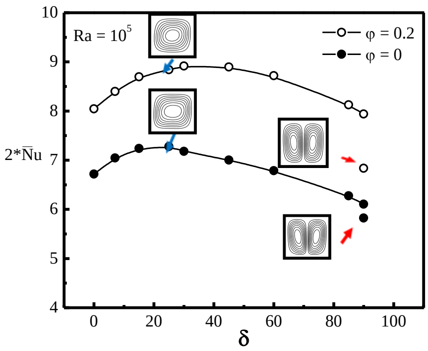


Figure 3: Variations of $\overline{\mathrm{Nu}}$ versus δ for $Ra = 10^5$ and φ = 0 (empty circles) and 0.2 (full circles).

corresponds to partial heating from below which leads to a multiplicity of solutions (monocellular and bicellular flows) where the bicellular flow is the less favorable to the heat transfer. In fact, the examination of the heatlines corresponding to the bicellular flow shows that the inactive area occupies a space comparable to the size of the convective cells and the heat exchange between the active elements occurs through a narrow vertical corridor between the cells. In the case of the monocellular flow, the nature of the flow (better contact between the cell and the active elements) favors the heat transfer from these elements to the rotating fluid. The effect of the inclination is amplified for the nanofluid leading to an important enhancement in heat transfer when compared to the pure fluid. More precisely, for the considered value of Ra, the minimum and the maximum enhancements observed, compared with the case of pure fluid, are respectively 19% and 30%, obtained for $\delta = 0°$ and $\delta = 90°$ **(**comparisons relative to monocellular flow**)**. Thus, the combined effects of the inclination and the volume fraction of nanoparticles could be a possible alternative to improve heat transfer.

3.3 Effect of the nanoparticles

The effect of the volume fraction of nanoparticles on mean Nusselt number is illustrated in Fig. 4 for different values of Rayleigh number. This figure shows that the heat transfer increases almost monotonically by increasing the volume fraction for all Rayleigh numbers with relatively important positive slopes in the dominating conductive ($Ra = 10^3$) and convective ($Ra > 10^5$) regimes. For $Ra = 10^4$, the mean Nusselt number is almost insensitive to the increase of the volume fraction of the nanoparticles since, in this intermediate regime, the negative impact of the nanoparticles on the flow intensity (see the velocity profile in Fig. 5a) and also on the manifest decrease of the temperature gradient (see Fig. 5b) are barely compensated by the positive effect of the conductivity without denying the effect of viscosity, which also plays an important role in the exchange process. Thus, the heat transfer process is controlled by complex interactions where the thermal diffusivity and the nanofluid viscosity coupled with gravity effect play a key role.

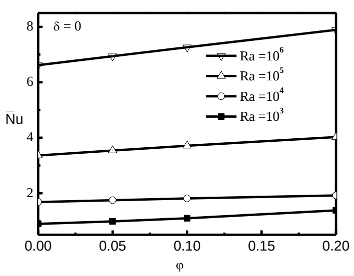
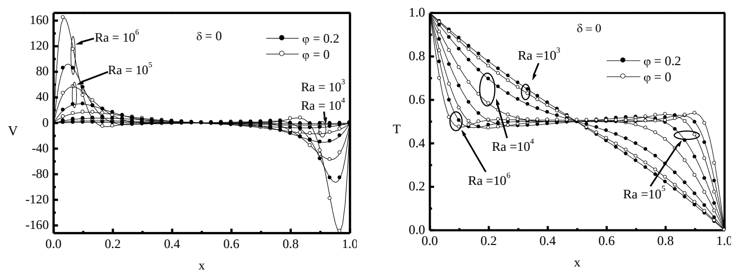


Figure 4: Variations of $\overline{Nu}$ versus φ for various values of Ra



(a) (b)

Figure 5 : Velocity (a) and temperature (b) profiles at mid-height of the enclosure for various Ra.

Conclusion

A numerical study using the LBM has been performed to investigate natural convection in a square inclined cavity partially heated and filled with CuO-water nanofluid. The main points drawn from the obtained results are the cooperating effects of the added CuO nanoparticles in the base fluid and the inclination of the cavity in the enhancement of heat transfer with maximum improvements obtained for specific inclinations which are depending on the volume fraction of the nanoparticles and Rayleigh number.

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