16èmes Journées Internationales de Thermique (JITH 2013)
Marrakech (Maroc), du 13 au 15 Novembre, 2013

On the search of optimal species separation process in an inclined Darcy-Brinkman porous cavity saturated with a binary mixture

A. Rtibi, M. Hasnaoui* and A. Amahmid,
Faculty of Sciences Semlalia, LMFE, Unit affiliated to CNRST (URAC 27)
BP 2390, Marrakesh, Morocco
* *Corresponding author: hasnaoui@uca.ma*

Abstract: Soret convection induced in an inclined rectangular porous cavity filled with a binary mixture and subjected to a constant heat flux is studied analytically and numerically using the Darcy-Brinkman model with the Boussinesq approximation. The relevant parameters for the problem are the thermal Rayleigh number ($R_T = 1\ to\ 10^6$), the Lewis number ($Le = 10$), the inclination angle of the cavity ($\theta = 0°\ to\ 180°$), the separation parameter ($\varphi = 0.5$), the Darcy number ($Da = 10^{-5}\ to\ 10^3$), and the aspect ratio of the cavity ($A_r = 12$). The limiting cases (Darcy and pure fluid media) are covered in this study. Optimum conditions leading to maximum separation of species are determined while varying the governing parameters in their respective ranges.

1. Introduction

The species separation process is a result of thermo-gravitational diffusion since a permanent applied thermal gradient engenders a gradient of concentration by Soret effect even in a medium with initially constant concentration distribution. Consequently, starting from a mixture of homogeneous composition, the coupling of the latter transport phenomena leads in some optimum conditions to lateral separation of the components. Recently, the subject of separation improvement in fluid mixtures is receiving increasing attention and efforts by leading researchers in the field. In this frame, some researchers teams [1] planned even to carry out experiments under microgravity conditions in order to minimize convection effect which prevents the separation process. In a previous investigation, Platten et al. [2] examined the effect of the inclination on separation in a thermogravitational column heated from the top. They concluded that separation can be substantially increased by choosing an optimal inclination of the column. The separation of the species of a binary mixture in the classical configuration of Rayleigh-Bénard (horizontal cell heated from below) was investigated by Elhajjar et al. [3]. They showed that the separation inside the horizontal cell may produce the same degree of separation but with a greater quantity of each species compared to a thermogravitational vertical column. Other authors, adopted a partitioning technique in order to improve separation within a vertical annular porous cylinder (Bennacer et al. [4]). They reported that the separation capability increases with the partitioning number. The influence of the porous matrix thermal properties on the separation rate in a model of packed thermogravitational column saturated by a binary mixture was studied by Davarzani and Marcoux [5]. Melnikov and Shevtsova [6] studied components separation due to the Soret effect in a system consisted of a fluid medium adjacent to a porous one with positive Soret coefficient. It is demonstrated that the presence of free liquid volumes near the lateral walls strongly affect the process of separation in the porous medium.

In the present study, the main objective consists to identify optimal conditions leading to maximum separation of species by studying the combined effect of the cavity inclination and the remaining controlling parameters (Darcy number, thermal Darcy-Rayleigh number and Soret parameter) on the thermodiffusion induced in a tall porous cavity inclined with respect to the horizontal. The extended Darcy-Brinkman model was used in the mathematical formulation of the problem. Numerical and analytical results are presented in terms of ΔC (difference characterizing the separation of species); they show that maxima of separation could be obtained for optimum combinations of Da, R_T and θ. The importance of the results obtained (good separation of species) lies in the fact that the process of separation could be observed even at high R_T (the inclination of the cavity being playing a damping role on the flow strength) for which experiments could be conducted easily compared to the case of low R_T.

Key Words: Separation of species; Soret effect; Tall porous cavity; Analytical and numerical study.

2. Problem formulation

The system under investigation is a two-dimensional Darcy-Brinkman porous cavity of length L′ and height H′, tilted at an angle θ with respect to the horizontal and bounded by four walls impermeable to mass

transfer. The long-side walls of the porous cavity are exposed to uniform fluxes of heat, q′, while its short walls are adiabatic. The fluid that saturates the porous medium is assumed homogeneous, isotropic and modeled as a Boussinesq-incompressible fluid whose density varies as $\rho = \rho_0\left(1 - \beta_T(T^{'} - T_0^{'}) - \beta_S(S^{'} - S_0^{'})\right)$, where ρ_0 is the density at temperature $T^{'} = T_0^{'}$ and concentration $S^{'} = S_0^{'}$ and $\beta_T$and $\beta_S$are the thermal and solutal expansion coefficients, respectively.
By neglecting the Dufour effect and using the vorticity-stream function formulation, the dimensionless governing equations in the stationary regime are presented as follows:

$$\xi = R_T(cos\theta\, \partial/\partial x - sin\theta\, \partial/\partial y)(T + \varphi S) + Da\nabla^2\xi \tag{1}$$

$$\partial T/\partial t + u\, \partial T/\partial x + v\, \partial T/\partial y = \nabla^2 T \tag{2}$$

$$\varepsilon\, \partial S/\partial t + u\, \partial S/\partial x + v\, \partial S/\partial y = (1/Le)(\nabla^2 S - \nabla^2 T) \tag{3}$$

The boundary conditions for the above equations are:

$$\left.\begin{matrix} x = \pm A_r/2 : \Psi = 0, \partial T/\partial x = 0\ and\ \partial S/\partial x = 0 \\ y = \pm 1/2 : \Psi = 0\ and\ \partial T/\partial y = \partial S/\partial y = -1 \end{matrix}\right\} \tag{4}$$

The governing parameters appearing in the dimensionless equations are the separation parameter, φ, the thermal Darcy-Rayleigh number, R_T, the Lewis number, Le, the cavity aspect ratio, A_r , the effective Darcy number, *Da*, and the inclination angle of the cavity, θ.

3. Analytical solution

The analytical solution is developed for steady state parallel flows, induced in slender cavities ($A_r >> 1$) under specific thermal boundary conditions. The presence of parallel flow solution was verified numerically but not presented here for reason of brevity. The behaviors observed allow the following simplifications:

$$\Psi(x,y) = \Psi(y), T(x,y) = C_T x + \theta_T(y)\ and\ S(x,y) = C_S x + \theta_S(y) \tag{5}$$

Where CT and CS are unknown constant temperature and concentration gradients respectively in the x direction. They are determined by imposing zero heat and mass fluxes across any transversal section of the cavity. The solution of the simplified governing equations, satisfying the boundary conditions in the y-direction depends on the sign of the following groups: ω^2,$(1/4\mathrm{Da}^2)(1 - 4\mathrm{Da}\omega^2)$, $\Omega_\mathrm{p}^2 = (1/2\mathrm{Da})\left(1 + \sqrt{1 - 4\mathrm{Da}\omega^2}\right)$ et $\Omega_m^2 = (1/2Da)\left(1 - \sqrt{1 - 4Da\omega^2}\right)$.

Hence, different cases should be considered. When $4Da\omega^2 \geq 1$, the solution of the problem becomes:

$$\Psi(y) = G + A_1 cosh\Omega_p y cos\Omega_m y + A_2 sinh\Omega_p y sin\Omega_m y \tag{6}$$

$$T(x,y) = C_T x + (C_T G - 1)y + C_T\left(A_1 I_1(\Omega_p, \Omega_m, y) + (A_2 + A_1\Omega_m/\Omega_p) I_4(\Omega_p, \Omega_m, y)\right) \tag{7}$$

$$S(x,y) = C_S x + ((C_T + LeC_S)G - 1)y + (C_T + LeC_S)\left(A_1 I_1(\Omega_p, \Omega_m, y) + (A_2 + A_1\Omega_m/\Omega_p) I_4(\Omega_p, \Omega_m, y)\right) \tag{8}$$

Where:

$$A_1 = \frac{-G\left(\Omega_p cosh\,\Omega_p/2\, sin\,\Omega_m/2 + \Omega_m sinh\,\Omega_p/2\, cos\,\Omega_m/2\right)}{\Omega_p sin\,\Omega_m/2\, cos\,\Omega_m/2 + \Omega_m sinh\,\Omega_p/2\, cosh\,\Omega_p/2}$$

$$A_2 = \frac{G\left(\Omega_p sinh\,\Omega_p/2\, cos\,\Omega_m/2 - \Omega_m cosh\,\Omega_p/2\, sin\,\Omega_m/2\right)}{\Omega_p sin\,\Omega_m/2\, cos\,\Omega_m/2 + \Omega_m sinh\,\Omega_p/2\, cosh\,\Omega_p/2}$$

$$\Omega_p = \left(1/\sqrt{4Da}\right)\sqrt{\sqrt{4Da\omega^2} + 1}$$

$$\Omega_m = \left(1/\sqrt{4Da}\right)\sqrt{\sqrt{4Da\omega^2} - 1}$$

$$
\begin{aligned}
C_T = {} & G + 2A_1\left[I_1(\Omega_p,\Omega_m,1/2) + (\Omega_m/\Omega_p)I_4(\Omega_p,\Omega_m,1/2)\right] + 2A_2I_4(\Omega_p,\Omega_m,1/2) \\
& - C_T\Big[(G^2 + A_1^2/4 - A_2^2/4) + 4G\left[A_1I_1(\Omega_p,\Omega_m,1/2) + \left[(\Omega_m/\Omega_p)A_1 + A_2\right]I_4(\Omega_p,\Omega_m,1/2)\right] \\
& + A_1A_2I_4(2\Omega_p,2\Omega_m,1/2) \\
& + (A_1^2/2 - A_2^2/2)\left[I_1(2\Omega_p,2\Omega_m,1/2) + (\Omega_m/\Omega_p)I_4(2\Omega_p,2\Omega_m,1/2)\right] \\
& + (A_1^2/4 + A_2^2/4)\left[I_2(2\Omega_m) + I_3(2\Omega_p)\right]\Big]
\end{aligned}
$$

$$
\begin{aligned}
C_S = {} & C_T + Le\Big[G + 2A_1\left[I_1(\Omega_p,\Omega_m,1/2) + (\Omega_m/\Omega_p)I_4(\Omega_p,\Omega_m,1/2)\right] + 2A_2I_4(\Omega_p,\Omega_m,1/2) - (C_T \\
& + LeC_S)\Big[(G^2 + A_1^2/4 - A_2^2/4) \\
& + 4G\left[A_1I_1(\Omega_p,\Omega_m,1/2) + \left[(\Omega_m/\Omega_p)A_1 + A_2\right]I_4(\Omega_p,\Omega_m,1/2)\right] + A_1A_2I_4(2\Omega_p,2\Omega_m,1/2) \\
& + (A_1^2/2 - A_2^2/2)\left[I_1(2\Omega_p,2\Omega_m,1/2) + (\Omega_m/\Omega_p)I_4(2\Omega_p,2\Omega_m,1/2)\right] \\
& + (A_1^2/4 + A_2^2/4)\left[I_2(2\Omega_m) + I_3(2\Omega_p)\right]\Big]\Big]
\end{aligned}
$$

$I_1(\Omega_p,\Omega_m,y) = (1/\Omega_p)sinh\Omega_p y cos\Omega_m y$, $\quad I_2(\Omega_m) = (2/\Omega_m)sin\,\Omega_m/2$, $\quad I_3(\Omega_p) = (2/\Omega_p)sinh\,\Omega_p/2$, $I_4(\Omega_p,\Omega_m,y) = \left(1/(\Omega_p^2 + \Omega_m^2)\right)\left[\Omega_p cosh\Omega_p y sin\Omega_m y - \Omega_m sinh\Omega_p y cos\Omega_m y\right]$.

4. Results and discussions

As the present problem is controlled by several dimensionless groups, some of these groups are maintained constant ($Ar = 12, Le = 10$ and $\varphi = 0.5$) and the analysis is limited to the effect of *Da*, R_T and θ on the separation, ΔC, defined as the difference of the mass fraction of the denser species between the two ends of the cavity (i.e. between x = 0 and x = A_r). The phenomenon of species separation counts among the main desired applications in the presence of Soret effect. It is well known that the separation of species is not possible when the convective regime is prevailing. Besides, this phenomenon occurs when the fluid is provided with weak circulation engendered by specific combinations of the governing parameters including the aspect ratio of the enclosures (tall or shallow cavities in general). Separation occurring at low Rayleigh numbers is facing technical realizations; difficulties associated with maintaining a small temperature difference between the active walls. In the present study, we search eventual optimal conditions leading to maximum separation even at relatively high Rayleigh numbers; the inclination of the cavity is expected to play an important role by reducing the convection effect. Hence, the attention is focused on the search of eventual specific inclinations of the cavity able to counter-act the gravity effect by bringing back (slowing down) the fluid flow to the threshold ensuring maximum separation of species.

The results obtained for species separation in the case of Darcy medium (*Da* =10^{-5}) are presented in Fig. 1b in terms of ΔC variations versus θ for various R_T. As expected, for R_T = 1, there exists a wide range of θ (30° ≤ θ ≤ 120°) for which the separation is maximum (ΔC ≈ 0.5). This means that, in this range of θ, the intensity of the flow is optimum (Ψ_C ≈ 0.15, Fig. 1a), which leads to an enhancement of the separation process. By increasing R_T, the behavior changes radically due to the strength of convection. In fact, in the whole range of θ, the maximum of separation is observed for a specific value of θ (θ = 163°) for $R_T \geq 200$ and its value (ΔC_{max} ≈ 0.53) is visibly higher than that observed for R_T = 1 and practically independent of R_T. In this separation process, the gravity plays a damping role since it weakens the role of convection by reducing it, for θ = 163°, to an optimum level that allows the obtaining of maximum separation. The importance of such behavior (maximum of species separation) occurring at relatively high R_T is due to the fact that it is easier in practice to perform experiments with noticeable temperature difference between the thermally active walls.

In the case of pure fluid medium, recovered with the Brinkman model for *Da* = 10, the examination of Fig. 2b shows that, two different tendencies of species separation are observed depending on R_T. In fact, it can be seen from Fig. 2b that, for $R_T = 10^3$, a relatively good separation (optimal coupling between thermodiffusion and convection) is observed in a wide range of θ (14° ≤ θ ≤ 139°) with a slight dip between these two limits leading to a relative minimum of ΔC, observed around θ ≈ 72°. But globally, in this range of θ, ΔC varies by about 15% (ΔC_{min}≈ 0.38 and ΔC_{max}≈ 0.45). By increasing R_T, the convection effect is promoted (Fig. 2a), and its negative impact on the separation prevails as long as the intensity of the fluid flow is higher than some threshold which should ensure an optimum coupling between convection and thermodiffusion. Thus, the inclination of the cavity plays a determinant role in ensuring this optimum coupling which finally emerges for θ around 163° at high values of R_T, leading to ΔC_{max} ≈ 0.53 for $R_T = 10^6$. The importance of these results (good separation of species) stems from the fact that they are obtained at high R_T for which experiments are conducted easily compared to the case of low R_T.

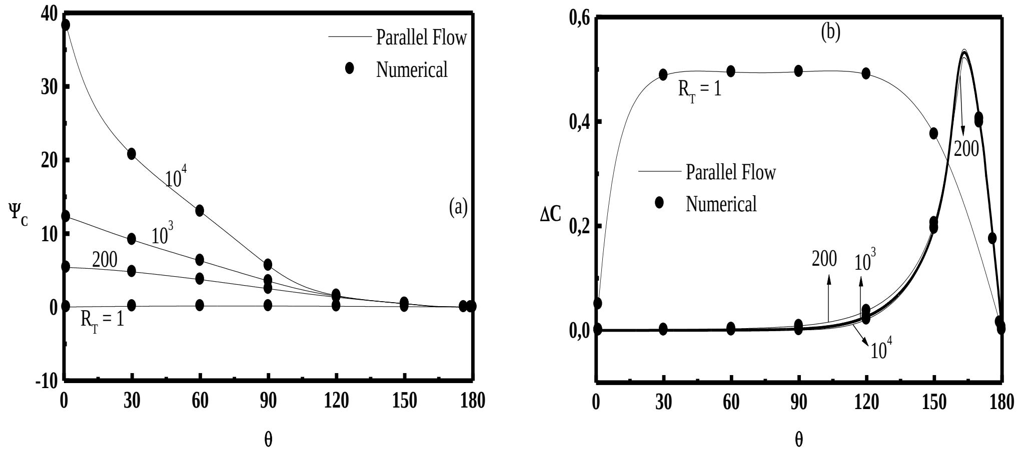

Figure 1: Effect of θ on Ψc (a) and ΔC (b) for Le = 10, *Da* = 10-5 and φ = 0.5.

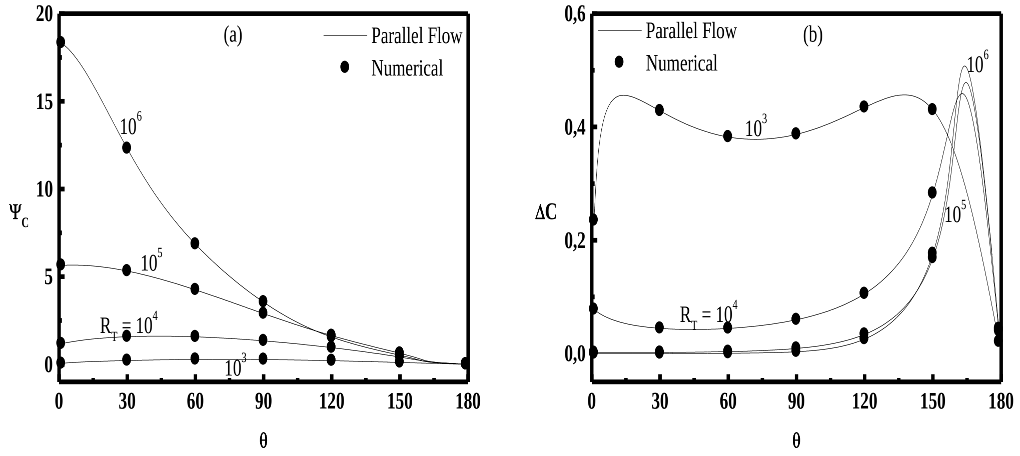

Figure 2: Effect of θ on Ψc (a) and ΔC (b) for Le = 10, *Da* = 10 and φ = 0.5.

5. Conclusion

Thermodiffusion effect on fluid flow and heat and mass transfers induced by Soret convection in an inclined Darcy-Brinkman porous layer subject to transverse gradients of temperature is studied analytically and numerically. The study is conducted in the case where the boundaries of the porous cavity are impermeable to mass transfer (pure Soret effect). The Darcy medium and pure fluid medium are covered by the present study as limiting cases for small and high Darcy numbers, respectively. For Le =10 and φ = 0.5, the existence of specific ranges of θ (depending on R_T and *Da*) in which the separation reaches its maximum are determined.The separation of species is observed when the convection is weakened by conjugate effects of R_T, *Da* and θ leading to optimum coupling between thermodiffusion and convection by bringing down the flow intensity to limits ensuring this optimum coupling.

Nomenclature

A_r aspect ratio of the porous matrix, L'/H', *m*
Da effective Darcy number, $K\mu_e/\mu H'^2$
D mass diffusivity of species, m^2/s
D_T thermo-diffusion coefficient, m^2/s
D_{eff} effective mass diffusivity, $\varepsilon' D$, m^2/s
g gravitational acceleration
H' height of the enclosure
K permeability of the porous medium
L' width of the porous layer
Le Lewis number, α/D
q' Constant heat flux per unit area, W/m^2
R_T thermal Darcy-Rayleigh number, $g\beta_T K \Delta T H'/(\alpha \upsilon)$
S dimensionless solute concentration, $(S' - S_0')/\Delta S'$
S_0 reference solute concentration $(at\ x = y = 0)$
$\Delta S'$ characteristic solute concentration, $-D_T S_0'(1 - S_0')\Delta T'/D_{eff}$
T dimensionless temperature, $(T' - T'_0)/\Delta T'$
T_0 reference temperature $(at\ x = y = 0)$, K
$\Delta T'$ characteristic temperature, $H'q'/\lambda$, K
(u, v) dimensionless velocities, $u'H'/\alpha, v'H'/\alpha$, *m/s*
(x, y) dimensionless coordinates, $(x'/H', y'/H')$, *m*

Symbols

α thermal diffusivity, m^2/s
Ψ dimensionless stream function, Ψ'/α
β_S solutal expansion coefficient
β_T thermal expansion coefficient
ε' porosity of the porous medium
ϑ kinematic viscosity of the fluid, Kg/m.s
μ dynamic viscosity of the fluid, m^2/s
μ_e effective viscosity, m^2/s
ρ density of the fluid mixture
φ separation parameter; $\varphi = \beta_S \Delta S'/\beta_T \Delta T'$
$(\rho C)_F$ heat capacity of the fluid mixture, $Kgm^{-1}s^{-2}K^{-1}$
$(\rho C)_S$ heat capacity of the saturated porous medium, $Kgm^{-1}s^{-2}K^{-1}$
σ heat capacity ratio, $(\rho C)_F/(\rho C)_S$
ε normalized porosity, ε'/σ
ξ dimensionless vorticity, $\xi' H'^2/\alpha$

Superscript

' dimensional variable

Subscripts

maxrefers to maximum value
minrefers to minimum value
0 refers to a reference state
S refers to solutal
T refers to thermal

Références

[1] V. Shevtsova, IVIDIL experiment on board the ISS, Advances in Space Research, vol. 46, pp. 672–679, 2010.

[2] J. K. Platten, M.M. Bou-Ali and J. F. Dutrieux, Enhanced Molecular Separation in Inclined Thermogravitational Columns, *J. Phys. Chem. B*, vol. 107, pp. 11763-11767, 2003.

[3] B. Elhajjar, M. C. Charrier-Mojtabi, and A. Mojtabi, Separation of a binary fluid mixture in a porous horizontal cavity, Physical Review E, vol. 77, 026310, 2008.

[4] R. Bennacer, A. A. Mohamad and M. El Ganaoui, Thermodiffusion in porous media: Multi-domain constitutant separation, *Int. J. Heat Mass Transfer*, vol. 52, pp. 1725-1733, 2009.

[5] H. Davarzani and M. Marcoux, Influence of solid phase thermal conductivity on species separation rate in packed thermogravitational columns: A direct numerical simulation model, C.R. Mécanique, vol. 339, pp. 355–361, 2011.

[6] D.E. Melnikov and V.M. Shevtsova, Separation of a binary liquid mixture in compound system: Fluid–porous–fluid, *Acta Astronautica*, vol. 69, Issues 7-8, pp. 381–386, 2011.