Mixed Convection in Cylindrical Enclosure Similar to the CZ Configuration

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Abstract: The objective of this work is to make a numerical study of mixed laminar convection in a cylindrical cavity similar to the Czochralski configuration. The cavity is filled with a liquid metal (Pr = 0.011). One third of the radius of the upper disc consisting crystal temperature T_C, rotated at angular velocity Ω. The cavity is heated laterally at the temperature T_h ($T_h > T_c > T_{melting}$). The finite volume method was used for the numerical solution of the governing equations. The results were compared with those in the literature. The effect of Richardson number on the flow structure and the thermal field has been presented and discussed, for Ri = 0.1, 0.4, 0.8, 1 and 2. The effect of aspect ratio of the cavity is also taken into account (A = H / R_C). The obtained results are in a good agreement with those in literature and show that, the rate of convection heat transfer decreases with the increase of Richardson number.

Key word: Convection, Czochralski technique, finite volume method, mixed convection.

1. Introduction

The production of single crystals is an important issue for the optoelectronics industry and electronics [1]. Among the techniques developed there found the Czochralski (CZ), this technique is the most currently used to produce large single crystals. The first use of this technique was carried out in 1916 by Jan Czochralski. This was the selection of a single crystal of tin wire [2]. After the discovery of the transistor the launch of a first draft industrial crystal was made with Germanium [3], next the silicon semiconductors and III-V (GaAs, InP) and II-VI (CdTe). Several researchers interested in the study of convection during crystal growth under different conditions, to improve the quality of single crystals obtained. The effect of rotation of Crystal crucible and the fluid Prandtl number on convection has been shown in the work [4-5]. These factors were complicated by other researchers with the addition of several factors, such as Marangoni convection, magnetic field and radiation [6-9]. The objective of the present work is to make a numerical study of mixed convection in a cylindrical cavity similar to the Czochralski configuration.

2. Physical system and mathematical formulation

The physical system under consideration was shown on the Figure 1. The considered process was stationary and axisymmetric. The thermo physical prosperities of liquid metal were supposed constant. The aspect ratio A (H/R_C) = 2, the enclosure was heated laterally by $T_h > T_{melting}$ and contain liquid metal (Pr = 0.011). The Cristal cooled by $T_c < T_h$ and subjected to the rotation of disc Ω, were the bottom of the enclosure is maintained adiabatic, in this conditions; the approximation of Boussinsq was valid. By using the following characteristic parameters of the problem: $R = r/R_c$, $Z = z/R_c$, $U = u/\Omega R_c$, $V = v/\Omega R_c$, $P = p/\rho(\Omega R_c)^2$, $\theta = (T - T_C)/(T_h - T_C)$, for the radius , the velocity component, radial and axial, the pressure and the temperature respectively. The dimensionless equations governing the mixed convection are writing:

a) Continuity equation:

$$\frac{1}{R}\frac{\partial}{\partial R}(RU) + \frac{\partial V}{\partial Z} = 0 \qquad (1)$$

b) Momentum equation in radial direction:

$$U\frac{\partial U}{\partial R} + V\frac{\partial U}{\partial Z} - \frac{W^2}{R} = -\frac{\partial P}{\partial R} + \frac{1}{\mathrm{Re}}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial U}{\partial R}\right) + \frac{\partial^2 U}{\partial Z^2} - \frac{U}{R^2}\right] \qquad (2)$$

c) Momentum equation in axial direction:

$$U\frac{\partial V}{\partial R} + V\frac{\partial V}{\partial Z} = -\frac{\partial P}{\partial Z} + \frac{1}{\mathrm{Re}}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial V}{\partial R}\right) + \frac{\partial^2 V}{\partial Z^2}\right] + Ri.\theta \qquad (3)$$

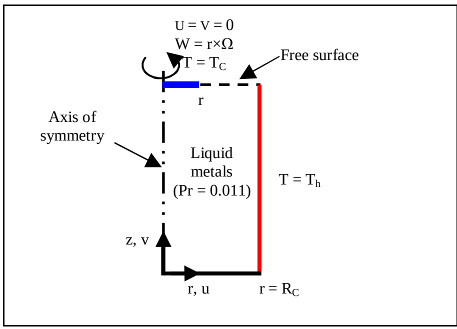


Figure 1: Physical configuration.

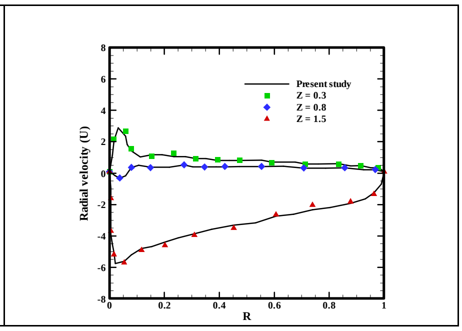


Figure 2: Comparison of our results with those of Li et al. [11]: Profile of radial velocity component U, at different altitudes, Z = 0.3, 0.8 and 1.5.

d) Momentum equation in Swirl direction:

$$U\frac{\partial W}{\partial R}+V\frac{\partial W}{\partial Z}+\frac{UW}{R}=\frac{1}{\mathrm{Re}}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial W}{\partial R}\right)+\frac{\partial^2 W}{\partial Z^2}-\frac{W}{R^2}\right] \quad (4)$$

e) Energy equation:

$$U\frac{\partial \theta}{\partial R}+V\frac{\partial \theta}{\partial Z}=\frac{1}{\mathrm{Re}.\mathrm{Pr}}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \theta}{\partial R}\right)+\left(\frac{\partial^2 \theta}{\partial Z^2}\right)\right] \quad (5)$$

The obtained dimensionless number are given as: Reynolds number (Re), Richardson number (Ri), Grashof number (Gr) and Prandtl number (Pr). The boundary conditions are:

a) Axis of symmetry:

$$R = 0 \text{ and } 0 \le Z \le 2\text{: } U = W = 0,\ \frac{\partial V}{\partial R}=0,\ \frac{\partial \theta}{\partial R}=0 \quad (6a)$$

b) Lateral wall of the enclosure:

$$R = 1 \text{ and } 0 \le Z \le 2 : U = V = W = 0,\ \theta = 1 \quad (6b)$$

c) The crystal:

$$0 \le R \le 1/3 \text{ and } Z = 2\text{: } U = V = 0,\ W = \Omega R,\ \theta = 0 \quad (6c)$$

d) Free surface:

$$1/3 \le R \le 1,\ Z = 2\text{: } \frac{\partial U}{\partial Z}=V=W=0,\ \frac{\partial \theta}{\partial Z}=0 \quad (6d)$$

e) Bottom wall of the enclosure:

$$0 \le R \le 1 \text{ and } Z = 0\text{: } U = V = W = 0,\ \frac{\partial \theta}{\partial Z}=0 \quad (6e)$$

The equations (1) − (5) associated with the boundary conditions (6a-e) are solved by a finite volume method (Patankar [10]), The SIMPLER Algorithm was used for a coupled velocity-pressure treatment.

3. Results and discussion

3.1 Validation and comparison of our results

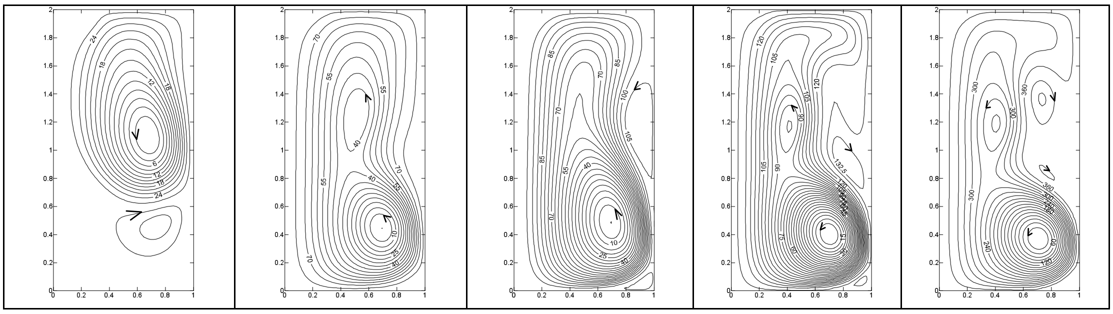
In order to give more confidence to the results of our numerical simulations, a comparison was made with the numerical results of Rong et al. [11]. The profiles of the radial velocity component U, for different altitudes, Z = 0.3, 0.8 and 1.5 are presented in Figure 2. We can see an excellent agreement between the two.

3.2 Effect of Richardson (Ri) number

To study the effect of Ri on the heat convection, we set the Reynolds number at Re = 1500 and we varied the Grashof number such that Ri have the following values: R = 0.1, 0.4, 0.8, 1 and 2. In Figures 3- 5, we have presented the streamlines, the isotherm line and the contours of the azimuthal velocity component for different Ri. A two-cell flow occurs for Ri = 0.1, the main cell rotates counter-clockwise and the other clockwise. From Ri = 0.4, we can see that the flow becomes single cell with small secondary cells in the corners of these cells which caused by the increase in the rate of fluid particles in the cavity.

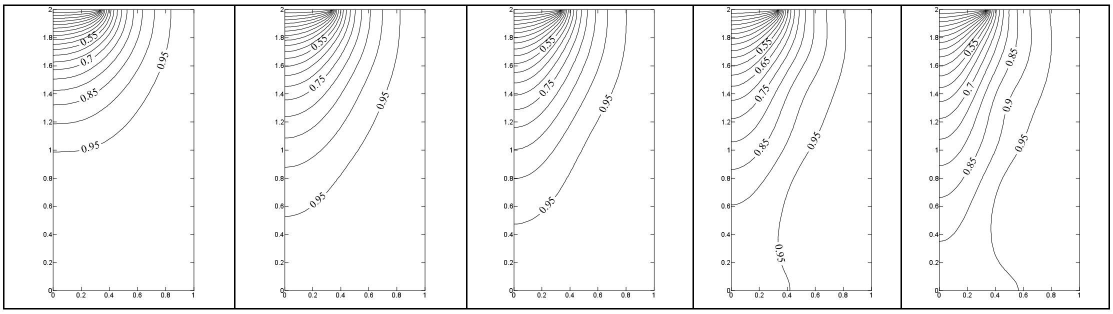
For the thermal field, we observed that the isothermal lines are nearly parallel to the isotherm line of the Crystal (1/3Rc), for all Ri number. This means that the mode of heat transfer is dominated by conduction. Increasing the curvature Ri increases and isothermal lines of point to the same azimuthal velocity, since the thermal convection strong with the increase of Ri. We also note that the variation of the axial velocity along the

Z axis (Figure 6) is negative, this shows that the direction of flow near the symmetry axis (R = 0) is in the opposite direction Z.



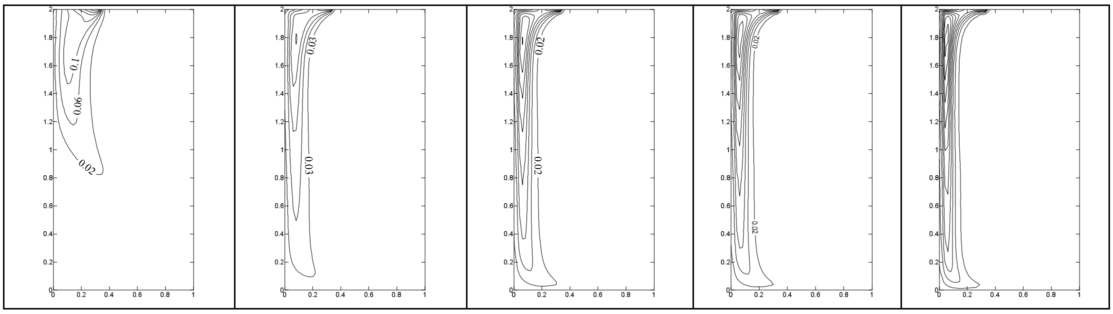
a)Ri = 0.1 b) Ri = 0.4 c) Ri = 0.8 d) Ri = 1 e) Ri = 2

Figure 3: Streamline for different Richardson number.



a)Ri = 0.1 b) Ri = 0.4 c) Ri = 0.8 d) Ri = 1 e) Ri = 2

Figure 4: Isothermal line for different Richardson number.



a)Ri = 0.1 b) Ri = 0.4 c) Ri = 0.8 d) Ri = 1 e) Ri = 2

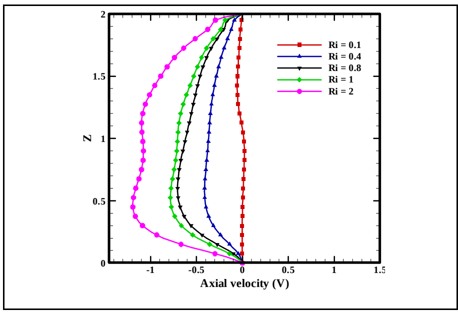
Figure 5: Streamline of azimuthal velocity component for different Ri.

The variation of average Nusselt number in function of Ri is shown in Figure 7. We can see that Nu decreases with increasing Ri, this allows us to conclude that the heat transfer improves as much as Ri decreased.

3.3 Effect of aspect ratio (A)

Four aspect ratios: A = 1/3, 1/2, 1, 2 and 3 were tested (Figure 8). The Grashof number and the Reynolds number are fixed respectively: $Gr = 10^6$ and Re = 2500.

For A < 1, the flow is unicellular and the central cell is moving toward the hot wall. However, for A> 1, the flow becomes two-cell, each cell flows to the inverse of the other. We also note that the streamline are inclined and extend according to the extension of the cavity. This means that the wall of the cavity has the effects on the structure of movement of fluid particles. For isothermal lines, we can see that they are almost parallel to the cold wall that means most of transfer in this region is by conduction. We can also see that the average Nusselt number increase with the increases of A to A = 1 (Figures 9-10), than decreases for A > 1 so to ensure a better heat transfer convection in Cz cavity, we can choose a square cavity (A = 1).



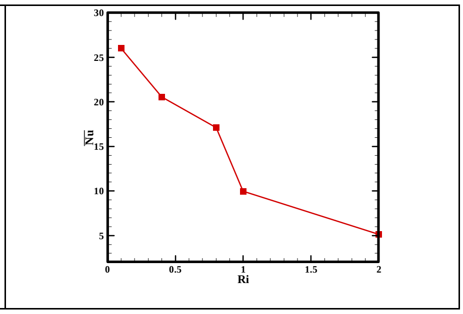


Figure 6: Profile of axial velocity component V, for different Richardson number.

Figure 7: Average Nusselt number in function of Richardson number.

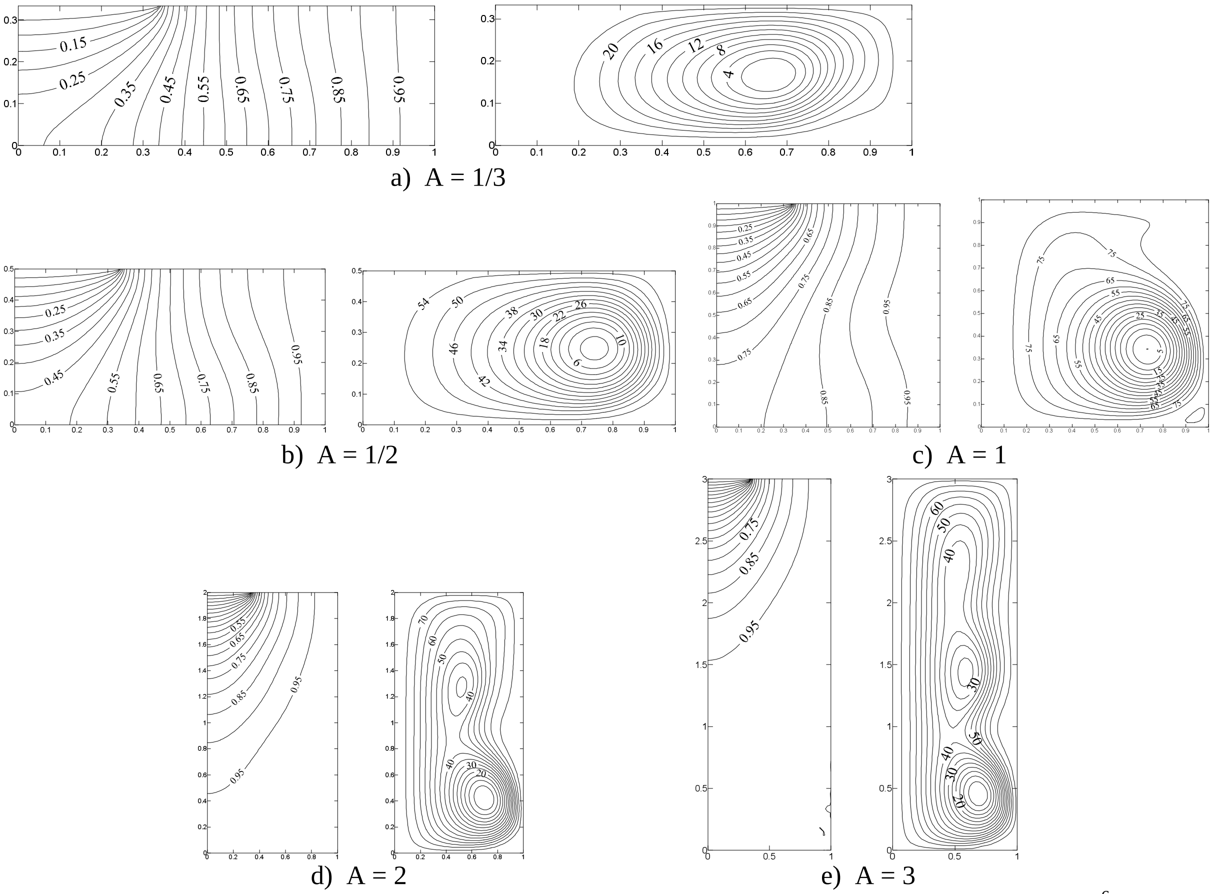
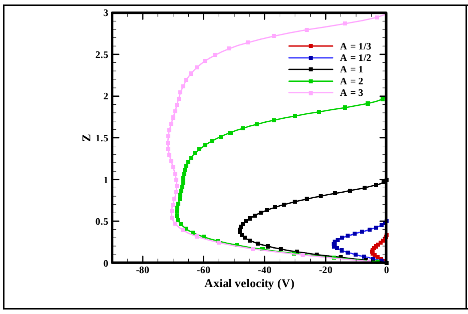


Figure 8: Isothermal ligne (right) and stream function (left), for different aspect ratio (Gr = 10^6).



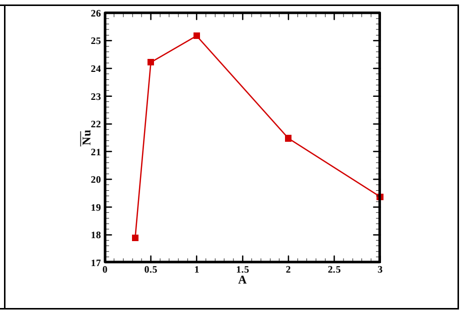


Figure 9: Profile of axial velocity component V, for different Richardson number.

Figure 10: Average Nusselt number in function of A.

Conclusion

A numerical study of mixed convection in the cavity similar to the Czochralski has been presented. Our results has been compared and validated with the available work and show that:

- The rate of heat transfer by convection decreases with the increase of Richardson number.
- The aspect ratio of the cavity has a great effect on the heat transfer rate.

The results presented in this work, allowing experimenters by the Czochralski method, prepare their processes and semiconductor single crystals in good thermal conditions.

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Nomenclature

A: Aspect ratio [-].
H: Height of the enclosure [m].
R_C: Radios of the cylinder [m].
R, Z: dimensionless coordinate [-].
U, V, W: dimensionless velocity component [-].
P: dimensionless pressure [-].
θ: dimensionless temperature [-].
$\overline{Nu}$: Average Nusselt number
g: Acceleration de la pesanteur [$m^2.s^{-1}$]
υ : Kinematic viscosity [$m^2.s^{-1}$].

Ω : Angular velocity [$rad.s^{-1}$]
ρ : Density [$kg.m^{-3}$]
β : Coefficient of thermal expansion [K^{-1}]
Re: Reynolds number $\mathrm{Re} = \Omega R_C^2 / \nu$
Ri: Richardson number $Ri = Gr / \mathrm{Re}^2$
Gr: Grashof number $Gr = g\beta(T_h - T_C)R_C^3/\nu^2$
Pr: Prandtl number $\mathrm{Pr} = \nu / \alpha$.