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Numerical Study of Double Diffusion Natural Convection in a Square Enclosure with Partially Active Vertical Wall

Abdennacer BELAZIZIA[1,2], Said ABBOUDI[1], Smail BENISSAAD[2]

[1]*Laboratory IRTES M3M, University of Technology, Belfort Montbéliard, Sévenans, 90010 Belfort Cedex, France*

[2]*Laboratoire d'Energétique Appliquée et de Pollution, Département de génie mécanique. Université Mentouri, Constantine, Algérie*.

nacer.belazizia@gmail.com

said.abboudi@utbm.fr

benissaad.smail@gmail.com

Abstract: In this work, we consider a two-dimensional numerical study of double diffusion natural convection in a square enclosure subjected to horizontal temperature and concentration gradients. The flow is driven by opposite thermal and solutal buoyancies. Finite volume method is used to solve the dimensionless governing equations. The active location tikes three positions in the left wall: top (T), middle (M) and bottom (B). The obtained results show that the increase of Ra_t leads to enhance heat and mass transfer rates. The flow is steady at: $Ra_t<7\ 10^4$ for (T) and $Ra_t<6\ 10^5$ for (M). The unsteady flow appears by the formation of regular (periodic) oscillations of particles in the flow when $Ra_t=7\ 10^4$ for top and $Ra_t=6\ 10^5$ for middle. While for case bottom, the flow is steady for high Rayleigh number ($Ra_t=10^8$). The fast Fourier transform has been used to determine the dominant period of oscillations. Which is (1/20) for (T) and (1/7.43) for (M).

Keywords **:** Thermosolutal convection, partially active wall, opposite buoyancies, unsteady flow, Instability.

1. Introduction

Natural convection in which the buoyant forces are due both to temperature and concentration gradients is generally referred to as thermosolutal convection or double-diffusive convection. Various modes of convection are possible depending on how temperature and concentration gradients are oriented relative to each other as well as to gravity: the stratified fluid can be subjected to horizontal or vertical temperature and concentrations gradients [1]. Natural convection in enclosures is investigated by many researches due to its wide application areas: thermal design of buildings, thermal energy storage systems, melting and solidification process, pollution dispersion in lakes and etc. Thermosolutal convection is also important in crystal growth processes. Heat and mass transfer through an enclosure is influenced by parameters such as wall boundary conditions, inclination, aspect ratio and cavity geometry. Double diffusive convection has been studied in many references [1-7].

In this work, we present a numerical study of thermosolutal natural convection in a square enclosure filled with a binary fluid ($CuSO_4+H_2SO_4+H_2O$ with $Pr=7$ and $Sc=240$) and submitted to horizontal temperature and concentration gradients. The main focus is on examining the effect of thermal Rayleigh number ($10^3 \leq Ra_t \leq 10^8$) on fluid flow. The unsteady oscillatory flow is also studied. The dominate frequency is determined by fast Fourier transform method. The rate of heat and mass transfer in the enclosure is measured in terms of the average Nusselt and Sherwood numbers.

2. Problem geometry

The geometry of the problem is shown in Fig. 1. The horizontal boundaries of the enclosure are impermeable and thermally insulated. The boundary conditions are selected in which to obtain opposing thermal and solutal buoyancy forces.

3. Governing equations

The flow in the enclosure is assumed to be two-dimensional. All fluid properties are constant. The fluid is considered to be incompressible and newtonian. The Boussinesq approximation is applied. Viscous dissipation, heat generation, radiation and Soret effects are neglected.

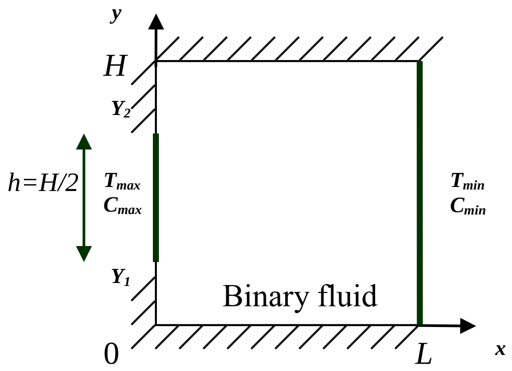


Figure. 1. Physical configuration.

- at $t=0$: $U=V=0$; $\theta=0$; $C=0$; $0\leq X\leq 1$, $0\leq Y\leq 1$
- for $t>0$:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (1)$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+P_r\nabla^2U \quad (2)$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+P_r\nabla^2V+P_r.Rat(\theta-NC) \quad (3)$$

$$\frac{\partial \theta}{\partial t}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\nabla^2\theta \quad (4)$$

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{P_r}{Sc}\nabla^2C \quad (5)$$

The boundary conditions in the dimensionless form are:

$U=V=0$, for all boundaries (6a)

$\theta=C=1$ for $X=0$, $Y\geq 1/2$ or $3/4\geq Y\geq 1/4$ or $Y\leq 1/2$ (6b)

$\theta=C=0$ for $X=1$, $0\leq Y\leq 1$ (6c)

$$\frac{\partial \theta}{\partial X}=\frac{\partial C}{\partial X}=0 \text{ for } Y=0 \text{ and } Y=1,\ 0\leq X\leq 1 \quad (6d)$$

The local Nusselt and Sherwood numbers are defined by:

$$Nu=-\frac{\partial \theta}{\partial X} \quad ; \quad Sh=-\frac{\partial C}{\partial X} \quad (7)$$

4. Numerical method

Finite volume method is used [8]. A uniform mesh is used in X and Y directions. A Hybrid scheme and first order implicit temporally discretization are used. A (60x60) grid was selected and used in all the computations. A comparison was made with the numerical results obtained by Nithyadevi [7]. A good agreement is obtained.

5. Results and discussions

The effect of thermal Rayleigh number on fluid flow, thermal and solutal fields is illustrated in Fig. 2. The hot active region is along the half portion of the left vertical wall. A single cell rotating in clockwise direction appears inside the enclosure. In consequence natural convection is dominated by thermal buoyancy (N=1 and $Le=\alpha/D$=34.28). By moving of the active location from the top to the bottom, we observe that the maximum absolute values of streamline function ψ and velocity flow $\boldsymbol{V}$ are more important. While the average Nusselt and Sherwood numbers are more important in the case (M). It is clear that the average Nusselt and Sherwood numbers are increasing with Rayleigh number as shown in Fig. 2. Heat and mass transfers are more important in the case (M). The fluid contained in the enclosure rises along the hot location and falls along the right cold wall, so thermal and concentration gradients are very important in these regions. For the three cases, concentration in the middle of the enclosure is almost constant. In consequence, we can notes that the position of the active location has a noticeable effect on the rate of heat and mass transfer and fluid velocity.

To know that the flow is steady or unsteady for each value of Ra_t, we fellow the temporary evolution of horizontal velocity U, at six locations, arbitrary selected in the fluid P_i (i=1:6) corresponding respectively to the points: (0.06, 0.49), (0.23, 0.83), (0.31, 0.14), (0.49, 0.49), (0.66, 0.83) and (0.83, 0.66). Figure 3 shows that the

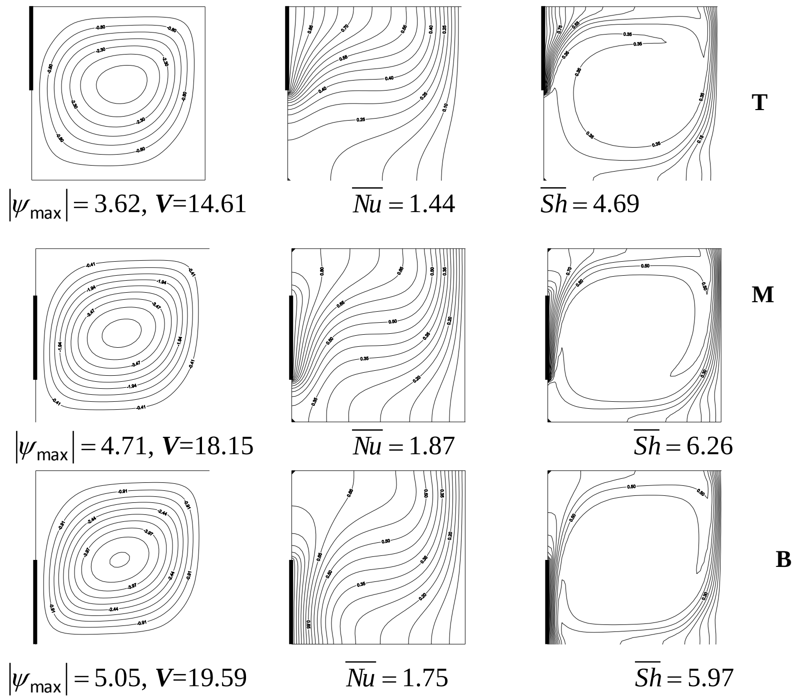


Figure. 2. Steady state of streamlines (left), isotherms (middle) and iso-concentrations (right) for Ra_t=10[4].

flow is steady in all points for the three cases: T (Ra_t=6.10[4]), M (Ra_t=5.10[5]) and B (Ra_t =10[7]). The same remark can be observed in fig. 4, which represents temporal evolution of the average Nusselt number along the active walls. Thermal balance is reached respectively at (t > 4) for T, (t > 1.5) for M and (t > 1.2) for B.

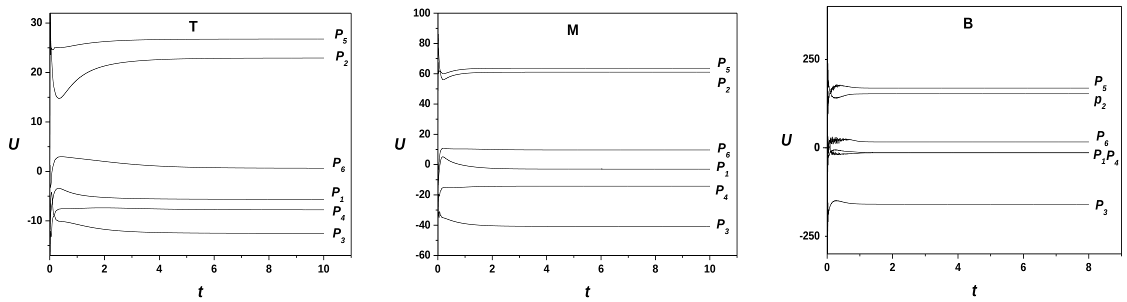


Figure. 3. Evolution of horizontal velocity for six points P_i (i=1:6). Ra_t=6.10[4].

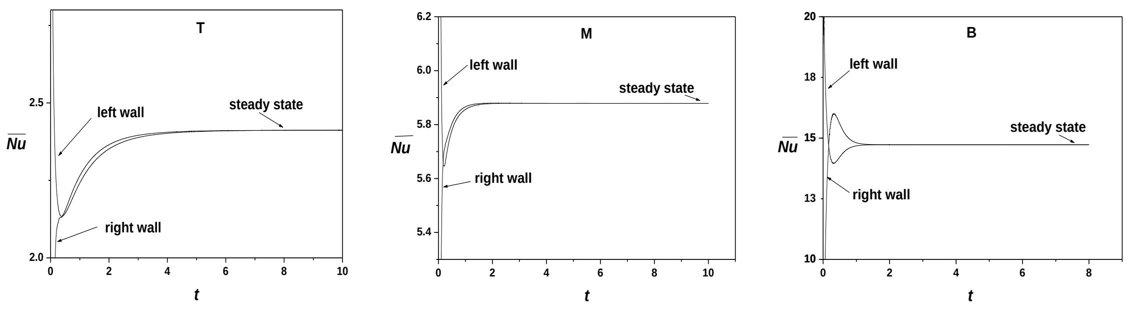
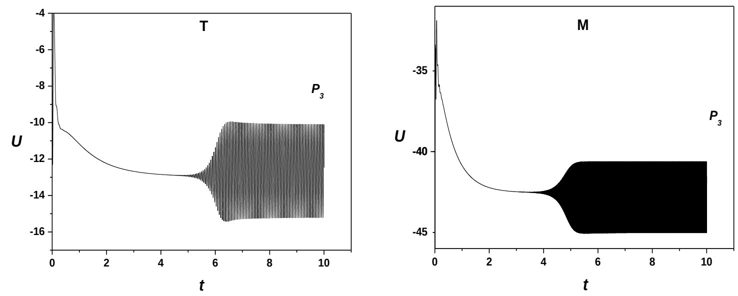


Figure. 4. Evolution of the average Nusselt number: top (T), middle (M) and bottom (B).

The unsteady solution can be known by determination of the critic thermal Rayleigh number Ra_{tcr}. In this case, we observe the formation of regular periodic oscillations of particles in the flow. In our problem, the steady solution is maintained until: $Ra_t=6.10^4$ for T and $Ra_t=5.10^5$ for M. The oscillatory unsteady one appears at $Ra_{tcr}=7.10^4$ and $Ra_{tcr}=6.10^5$ respectively for (T) and (M) as shown in Fig. 5**.**

In order to avoid the numerical perturbations, we reduce time step from 10^{-4} to 2.5 10^{-5}. The amplitude oscillations keep the same value. In consequence the obtained instability is a physical one. We note that the

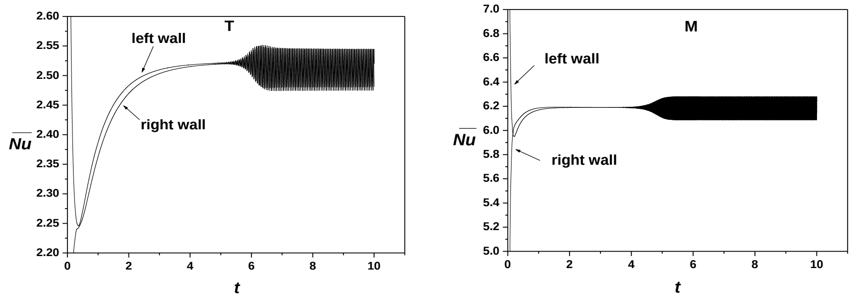
Figure. 5. Evolution of horizontal velocity U, for top and middle location in point P_3.



amplitude of the oscillations of each point P_i depends on its position with the walls enclosure.

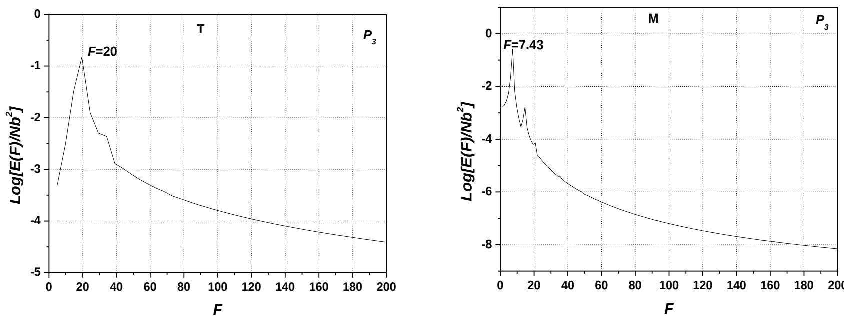
The rate of heat transfer in the enclosure for the two cases (T and M) are illustrated in Fig. 6. The temporary evolutions of the average Nusselt number along the active walls are also periodic around their average values respectively (2.51) for (T) and (6.2) for (M).

Figure. 6 Evolution of the average Nusselt number.



To obtain the energy spectrum of the oscillations, we use the fast Fourier transform (FFT) of certain number of values (N=2^{11}) corresponding to horizontal velocity. F_{cr} denotes the energy pic which is the dominant frequency. Fig. 7 shows the variation of energy perturbations with their frequency in P_3. We note that $F_{cr}=20$ for (T) and F_{cr} =7.43 for (M). F_{cr} is the same in the other points for the two cases (T and M). In consequence the dominant period are respectively 1/20 and 1/7.43.

Figure. 7. Energy spectrum corresponding at P_3.



Conclusion

A numerical study was employed to analyze the flow, heat and mass transfer of a binary fluid (chemical solution) filled in a square enclosure with top, middle and bottom active location of the left vertical wall. The flow is driven by opposite thermal and solutal buoyancies. The following conclusions are summarized. It is found that the rate of heat and mass transfer is more important in the case (M). The flow is steady until $Ra_t<7.10^4$ for (T) and Ra_t<6.10^5 for (M). The unsteady flow appears by the formation of regular (periodic) oscillations of particles in the flow when $Ra_t=7.10^4$ for (T) and Ra_t=6.10^5 for (M). The dominant periods of oscillations calculated with fast Fourier transformation are 1/20 for (T) and 1/7.43 for (M). For the case (B), the fluid flow continue to be more steady and stable without showing a particular oscillations, for high Rayleigh number (Ra_t=10^8).

Nomenclature

A	aspect ratio, H/L	D	solutal diffusivity, $m^2.s^{-1}$
C	dimensionless concentration (C^*-C_{min})/ΔC^*	E	spectral energy
		F	frequency of oscillations

g	gravitational acceleration, $m.s^{-2}$
Le	Lewis number, Le= α/D
Nu	local Nusselt number, Eq. (7)
$\overline{Nu}$	average Nusselt number, Eq. (8)
N	buoyancy ratio number, $(\beta_s \Delta C^*)/(\beta_t \Delta T)$
P	dimensionless pressure, $p/(\alpha/H)^2$
Pr	Prandtl number of the fluid, υ/α
Ra_t	thermal Rayleigh number, $g\beta_t H^3\Delta T/\upsilon\alpha$
Sc	Schmidt number, υ/D
Sh	local Sherwood number, Eq. (7)
$\overline{Sh}$	average Sherwood number, Eq. (8)
t	dimensionless time, $t^*/(H^2/ \alpha)$
U, V	dimensionless velocity components, u/(α/H), v/ (α/H)
V	dimensionless velocity of the flow
X, Y	non-dimensional cartesian coordinates, x/H, y/H

Greek symbols

α	thermal diffusivity, $m^2.s^{-1}$
β_t	thermal expansion coefficient, K^{-1}
β_s	solutal expansion coefficient, $m^3.Kg^{-1}$
θ	non-dimensional temperature, $(T-T_{min})/\Delta T$
υ	kinematics viscosity, $m^2.s^{-1}$
ρ	density, $Kg.m^{-3}$
ψ	non-dimensional stream function, $U=\partial \psi/ \partial Y$
ΔT	temperature difference, $(T_{max} - T_{min})$
ΔC^*	concentration difference, $(C_{max} - C_{min})$

Subscripts

$_0$	reference state
$_{max}$	maximum
$_{min}$	minimum
*	dimensional parameter

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