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Thermal field modeling of homogeneous rotating turbulence

Authors : Besma CHEBBI and Mounir BOUZAIANE
Adresses of authors: Laboratoire de Mecanique des Fluides, Département de Physique, Faculté des Sciences de Tunis
*Email of authors :**besma.chebbi@gmail.com**, bouzaianem@gmail.com*

Abstract : Thermal field of homogeneous sheared turbulence subjected to system rotation is examined using second order modeling approach. In this fact, the Launder-Reece-Rodi model, the Speziale- Sarkar-Gatski model and the Shih-Lumley model are retained to closure a set of equations describing the turbulent flow. So, non dimensional modeled equations are solved numerically with the fourth Runge-Kutta method for values sequence of non dimensional rotation number. Then, the closure models are evaluated by comparing simulation results with recent ones of direct numerical simulations. Firstly, the predictions of retained second order models have confirmed the existence of asymptotic equilibrium states. Furthermore, these results have demonstrated the advantage of using the Launder-Reece-Rodi and the Shih-Lumley models over the Speziale-Sarkar-Gatski model. Finally an important conclusion can be drawn that thermal field has been strongly affected by rotation.

Keywords :

Thermal field, rotating turbulence, modeling.

1. Introduction

Rotating turbulence can be found in many physical and industrial applications where rotation is accountable for fluid motion. Rotation is, in fact, detected in nature, especially oceanic current and atmospheric boundary as well as in industrial mechanical systems like turbomachines. As regards the dynamic of geophysical flows (oceanic or atmospheric), it is typically presented in meteorological or climatic studies for the atmosphere and waves or current studies for the ocean. As to turbomachines, there are systems that are liable to rotating motion setting of the flow.

In this context, rotation turbulence remains of the big interest subject to study. And by this right, several experimental, theoretical or numerical works have been presented to explore and to investigate turbulent rotating flows. Furthermore, second order modeling of rotating turbulent flow have constituted, since the last few years, a fundamental important purpose which offer the interesting alternatives and the promising progresses to describe and to analyze the kinematic field of homogeneous sheared turbulence submitted to the rotation.

While kinematic field received considerable attention by authers, as Porosova [1], Speziale et al. [2], the thermal field have been enough considered. So that, we are motivating, in this work, to explore and to examine the effects of rotation on the thermal field in homogeneous sheared turbulence. For this fact, the three sophisticated second order closure models of Launder-Reece-Rodi [3], Speziale-Sarkar-Gatski [4] and Shih-Lumley [5] are retained here for the pressure-strain correlation, the pressure-temperature correlation and the transport equations of the kinematic and thermal dissipations of respectively the turbulent kinetic energy and the variance of temperature. The results of Direct numerical simulations of Brethouwer [6], which are in our opinion the most interesting work of the considered flow, have been retained to evaluate the second order closure models. We are interesting essentially to look for the prediction of equilibrium asymptotic states in the first time and to perform the capacity of the second order models to predict this asymptotic behavior at long time of dimensionless thermal parameters under effect rotation in the second time.

In section 2, we will review the turbulent and the transport equations governing the flow and describing thermal rotating turbulence. Section 3 will describe the details of second order modeling. Section 4 will present numerical results and their discussion. Finally, in section 5, we will summarize our results and we will provide conclusions.

2. Governing equations

In this paper, some assymptions are considered for fluid flow that is incompressible, the physical properties are constant and turbulence is subjected to constant mean velocity gradient as a rate S and to uniform solid-body rotation as a rate Ω.

2.1. Turbulent equations

Evolution equations of turbulent quantities using tensor notation in a rotating frame are the Navier-Stokes equations and the energy equation.

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial u_i}{\partial t} + \overline{U_k}\frac{\partial u_i}{\partial x_k} = -\frac{\partial p}{\partial x_i} - u_k\frac{\partial \overline{U_i}}{\partial x_k} - 2\varepsilon_{imp}\Omega_m u_p + \frac{\partial}{\partial x_k}[\nu\frac{\partial u_i}{\partial x_k} - u_i u_k + \overline{u_i u_k}] \quad (2)$$

$$\frac{\partial \theta}{\partial t} + \overline{U_k}\frac{\partial \theta}{\partial x_k} = -u_k\frac{\partial \overline{T}}{\partial x_k} + \frac{\partial}{\partial x_k}[\alpha\frac{\partial \theta}{\partial x_k} - \theta u_k + \overline{\theta u_k}] \quad (3)$$

Where ν, α, δ_{ij} and ε_{imp} are kinematic viscosity, scalar diffusivity, the kronecker symbol and the alternating tensor. Here $\overline{U_k}$ and $\overline{T}$ are respectively the mean velocity and the mean temperature while u_k and θ are respectively the fluctuating velocity and the fluctuating temperature.

2.2. Transport equations

In this part, the evolution equations of second order moments are derived. In classical way, second order moments present a convective sight described by a transport equation. So, transport equations for the components $\overline{u_i u_j}$ of the Reynolds stress, the components $\overline{\theta u_i}$ of the turbulent heat flux, the turbulent kinetic energy K, and the variance of temperature $\overline{\theta^2}$ can be obtained from combination of different components of the momentum equation. Here, we are limited to present the transport equations describing thermal fields of turbulence.

$$\frac{d\overline{\theta u_i}}{dt} = -\overline{u_i u_k}\frac{\partial \overline{T}}{\partial x_k} - (S\delta_{i1}\delta_{k2} + 2\varepsilon_{imk}\Omega_m)\overline{\theta u_k} - \overline{\theta\frac{\partial p}{\partial x_i}} - (\nu+\alpha)\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial \theta}{\partial x_k}}$$
$$+\frac{\partial}{\partial x_k}[\nu\overline{\theta\frac{\partial u_i}{\partial x_k}} + \alpha\overline{u_i\frac{\partial \theta}{\partial x_k}} - \overline{u_i u_k \theta}] \quad (4)$$

$$\frac{d\overline{\theta^2}}{dt} = -2\overline{\theta u_k}\frac{\partial \overline{T}}{\partial x_k} - 2\alpha\overline{\frac{\partial \theta}{\partial x_k}\frac{\partial \theta}{\partial x_k}} + \frac{\partial}{\partial x_k}[\alpha\frac{\partial \overline{\theta^2}}{\partial x_k} - \overline{u_k\theta^2}] \quad (5)$$

3. Second order modeling

Transport equations of turbulent heat flux and variance of temperature can be written following this considered flow as:

$$\frac{d}{dt}\overline{\theta u_i} = P_{i\theta} + \phi_{i\theta} - \varepsilon_{i\theta} \quad (6)$$

$$\frac{d\overline{\theta^2}}{dt} = P_\theta - \varepsilon_\theta \quad (7)$$

Here terms denoted by P are terms of production due to mean kinematic and thermal gradients and they express interaction between mean and turbulent motion.

$$P_{i\theta} = -\overline{u_i u_k}\frac{\partial \overline{T}}{\partial x_k} - (S\delta_{i1}\delta_{k2} + 2\varepsilon_{imk}\Omega_m)\overline{\theta u_k} \quad (8)$$

$$P_\theta = -2\overline{u_i\theta}\frac{\partial \overline{T}}{\partial x_i} \quad (9)$$

Terms denoted by Ɛ are terms of dissipation due to molecular effects of viscosity and thermal conduction.

$$\varepsilon_{i\theta} = (\alpha + \nu)\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial \theta}{\partial x_k}} \quad (10)$$

$$\varepsilon_\theta = 2\alpha\overline{\frac{\partial \theta}{\partial x_k}\frac{\partial \theta}{\partial x_k}} \quad (11)$$

Finally, $\phi_{i\theta}$ is the term of pressure-temperature gradient correlation.

$$\phi_{i\theta} = -\overline{\theta\frac{\partial p}{\partial x_i}} \quad (12)$$

The nonlinearity of these terms give impossibility to integrate transport equations. So, to resolve this closure problem, we resort to the second order modeling [7]. This last one, represents the term of production as exact and considers principally pressure-temperature gradient correlation that is classically decomposed on two contributions:

$$\phi_{i\theta} = \phi_{i\theta}^s + \theta_{i\theta}^r \quad (13)$$

The first contribution is called slow part and it characterizes the nonlinear mechanism of interaction between turbulent fluctuations. The second contribution is called the rapid part. It is linear and bring to the fore a mechanism of interaction between turbulent and mean motions.
For thermal field of homogeneous shear turbulent flow subjected to the rotation, two of the common second order models are retained.

3.1. The Launder-Reece-Rodi model

For the pressure-temperature gradient correlation, slow and rapid parts are written separately as follows:

$$\phi_{i\theta}^s = -C_1'\frac{\varepsilon}{k}\overline{\theta u_i} \quad (14)$$

$$\phi_{i\theta}^r = 0{,}8\overline{\theta u_k}\,\overline{U}_{i,k} - 0{,}2\overline{\theta u_k}\,\overline{U}_{k,i} + R\overline{\theta u_k}\varepsilon_{ijk}(\overline{U}_{i,k} + \overline{U}_{k,i}) \quad (15)$$

3.2. The Shih-Lumley model

The slow and the rapid parts are written respectively in the following forms [8]:

$$\phi_{i\theta}^{s} = -\frac{\varepsilon}{\overline{q^2}}\phi^{\theta}\overline{\theta u_i} \quad (16)$$

$$\phi_{i\theta}^{r} = 2\overline{U_{j,k}}I_{ijk} \quad (17)$$

For ϕ^{θ} , it takes the following form :

$$\phi^{\theta} = \frac{\beta}{2} + r_c - \left(\frac{(\beta-2)II_d/6}{II_d/3 + b_{ij}d_{ij}^2 - b_{ij}d_{ij}}\right) \quad (18)$$

In this expression β is already given by:

$$\beta = 2 + \frac{8}{9}\{1 + 62,4(-II + 2,3III)\} \quad (19)$$

b_{ij} is the anisotropic tensor of Reynolds b and, II_d and d_{ij} will be in these forms :

$$d_{ij} = \frac{\overline{\theta^2}\,\overline{u_i u_j} - \overline{\theta u_j}\,\overline{\theta u_j}}{\overline{\theta^2}\,\overline{q^2} - \overline{\theta u_p}\,\overline{\theta u_p}} \quad \text{and} \quad II_d = \frac{d_{ii}^2 - d_{ij}^2}{2} \quad (20)$$

r_c is the characteristic time rate given by :

$$r_c = \frac{\overline{q^2}\varepsilon_{\theta}^2}{\varepsilon\overline{\theta^2}} \quad (21)$$

whereas I_{ijk} term is linearly function of $\overline{\theta u_i}$ and his form [9]:

$$I_{ijk} = 0,4\delta_{ij}\overline{\theta u_k} - 0,1(\delta_{ik}\overline{\theta u_j} + \delta_{\overline{jk}}\overline{\theta u_i}) + 0,1b_{ij}\overline{\theta u_k} - 0,3(b_{ik}\overline{\theta u_j} + b_{jk}\overline{\theta u_i}) + 0,2\delta_{ij}b_{kp}\overline{\theta u_p} \quad (22)$$

3.3. Nondimensional equations

These last models have solved to closure the evolution equations in which is associated modeled transport equation of temperature variance dissipation that take this form:

$$\frac{d\varepsilon_{\theta}}{dt} = -C_{d1}\frac{\varepsilon_{\theta}}{\overline{\theta^2}} - 2C_{d2}\frac{\varepsilon}{k}\varepsilon_{\theta} - C_{d3}\overline{\theta u_2}G - 2C_{d4}\frac{\overline{u_1 u_2}}{k}S\varepsilon_{\theta} \quad (23)$$

In order to describe the thermal field behavior, nondimensional parameters are introduced as: the ratio of thermal fluxes $\rho_1 = \frac{\overline{\theta u_1}}{\overline{\theta u_2}}$, the turbulent thermal correlation coefficient $\rho_2 = \frac{\overline{\theta u_1}}{\theta' u_1'}$ and the ratio $\rho_3 = \frac{\theta'/G}{q'/S}$.

Just now nondimensional equations can be written considering, for example, the classic model of Launder-Reece-Rodi. These equations are given by:

$$\frac{d\rho_1}{d\tau}=-\frac{1}{\rho_3}\frac{b_{12}}{(b_{11}+\frac{1}{3})^{\frac{1}{2}}}-C_1\frac{\varepsilon}{kS}\rho_1-\frac{\rho_1}{\rho_2}(1.8-3R)+\rho_1(1-2R)\frac{b_{12}}{b_{11}+\frac{1}{3}}+\frac{C_1}{4}\rho_1\frac{\varepsilon}{kS}\frac{b_{11}}{b_{11}+\frac{1}{3}}$$
$$+\frac{C_2}{4}(1-\frac{2}{3}C_2)\rho_1\frac{b_{12}}{b_{11}+\frac{1}{3}}+\frac{1}{6}\rho_1\frac{\varepsilon}{kS}\frac{1}{b_{11}+\frac{1}{3}}+\frac{\rho_1^2}{\rho_2\rho_3}+\frac{1}{2}\rho_1\frac{\varepsilon}{kS}r_c \quad (24)$$

$$\frac{d\rho_2}{d\tau}=-\frac{\rho_2}{\rho_1\rho_3}\frac{b_{12}}{(b_{11}+\frac{1}{3})^{\frac{1}{2}}}+3R-1.8+(R-0.2)\rho_2^2-\frac{\rho_2^2}{\rho_1\rho_3}\frac{b_{22}+\frac{1}{3}}{(b_{11}+\frac{1}{3})^{\frac{1}{2}}} \quad (25)$$

$$\frac{d\rho_3}{d\tau}=-\frac{\rho_1}{\rho_2}(b_{11}+\frac{1}{3})^{\frac{1}{2}}-\frac{1}{2}\rho_3\frac{\varepsilon}{kS}r_c+\rho_3 b_{12}+\frac{1}{2}\rho_3\frac{\varepsilon}{kS} \quad (26)$$

In the same manner, others nonlinear equations can be written for the thermal field of turbulence using the Shih-Lumley model.

4. Results and discussion

In this work, we are studying rotation effects on the evolution of thermal field of homogeneous sheared turbulence. Pecular attention will be accorded, here, to equilibrium asymptotic behavior of dimensionless parameters, we have started by modeling a set of equations governing considered flow. In this fact, two of the most used second order models are retained for the thermal field as Launder-Reece-Rodi and Shih-Lumley. But we have to precise that the speziale-Sarkar-Gatski (SSG) model has been retained, in addition to launder-Reece-Rodi (LRR) and Shih-Lumley (SL) models for the kinematic field, but also classic models for transport dissipation equations of respectively kinetic energy and temperature variance are furtheremore retained.

Hence we refer by model 1 to LRR models separately for the kinematic and thermal fields, by model 2 to SSG model for kinematic field plus SL model for thermal field, and by model 3 to SL models separately for kinematic and thermal fields.

On the second step, numerical integration for three nonlinear differential equations using the fourth order Runge Kutta method is approached. The numerical integration is carried out separately for the values 0, 0.25, 0.5, -0.25, -0.5, -0.75 of nondimensional rotation number R, right taking into account the initial conditions both of the Direct Numerical Simulation (DNS) of Brethouwer and the experience results of Tavoularis and Corrsin [10]. At long time evolution, a general tendency to equilibrium asymptotic states for dimensionless thermal parameters such as $\rho_1=\frac{\overline{\theta u_1}}{\overline{\theta u_2}}$, $\rho_2=\frac{\overline{\theta u_1}}{\theta' u_1'}$ and $\rho_3=\frac{\theta'/G}{q'/S}$ are observed on following tables.

Table 1: Equilibrium values predicted for ρ_1

	Model 1	Model 2	Model 3	DNS
R=0	-1.080	-1.120	-0.865	…
R=0.25	-0.283	-1.460	-0.240	…
R=0.5	…	-1.800	5.440	…

For the turbulent heat flux rate, the equilibrium values are observed in different cases according to models 2 and 3 on the contrary of the model 1 where absence of equilibrium values is observed for R=0.5. The effect of rotation can be felt on the dimensionless thermal parameter ρ_1. In fact, growth of ρ_1 is observed for the model 3 against a decrease of the same parameter for models 1 and 2.

Table 2: Equilibrium values predicted for ρ_2

	Model 1	Model 2	Model 3	DNS
R=0	0.963	0.297	0.616	0.800
R=0.25	0.461	0.621	0.311	0.472
R=0.5	…	…	-0.842	…

For the correlation coefficient ρ_2 , there is tendency to equilibrium states for almost all the cases. ρ_2 have been strongly affected by rotation; ρ_2 Is decreasing nearly the half time for models 1 and 2 in accordance with DNS results, whereas, it is increasing nearly the half time for model 2.

Table 3: Equilibrium values predicted for ρ_3

	Model 1	Model 2	Model 3	DNS
R=0	1.410	2.440	0.834	0.846
R=0.25	1.640	2.270	0.930	1.230
R=0.5	…	…	…	…

For the dimensional ratio ρ_3, equilibrium asymptotic values have been found for R=0and R=0.25where the thermal parameter is also affected by rotation with an increase of ρ_3 for models 1 and 3 in agreement with DNS results against a weakly decrease for model 2.

In term of evaluating turbulence models, time evolution of dimensionless parameters versus dimensionless time St for two different rotation numbers are represented. So in figures (1.a) and (1.b), the asymptotic equilibrium behavior is observed for each of three models. A clear agreement can be seen between models 1 and 2 with DNS results for $St\rangle 10$ for R=0, but a perfect accord is observed between models 1 and 3 in the way and DNS results in the other way for R=0.5.

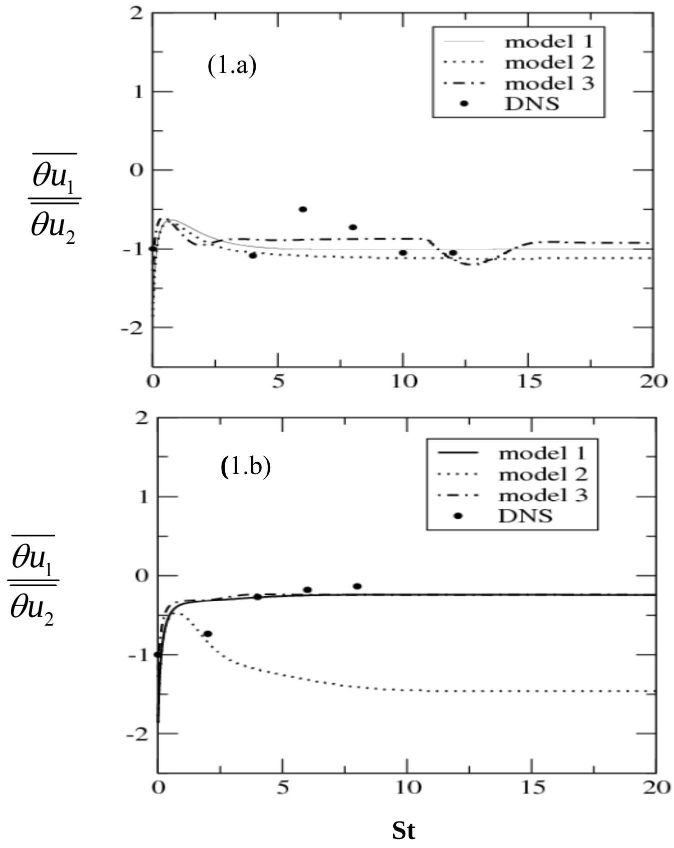

Figure 1 : time evolutions of the turbulent heat fluxes ratio for the three models in the cases R=0 (1.a) and R=0.25 (1.b)

In figures (2.a) and (2.b), agreement with DNS results can be seen on the time evolution of correlation coefficient according to model 1 for $St\rangle 4$ in the case R=0 and for $St\rangle 5$ in the case R=0.25. On figure (2.b), equilibrium state tendency is peculiarly observed, at long time evolution, with a strong nearly concordance with models 1 and 3.

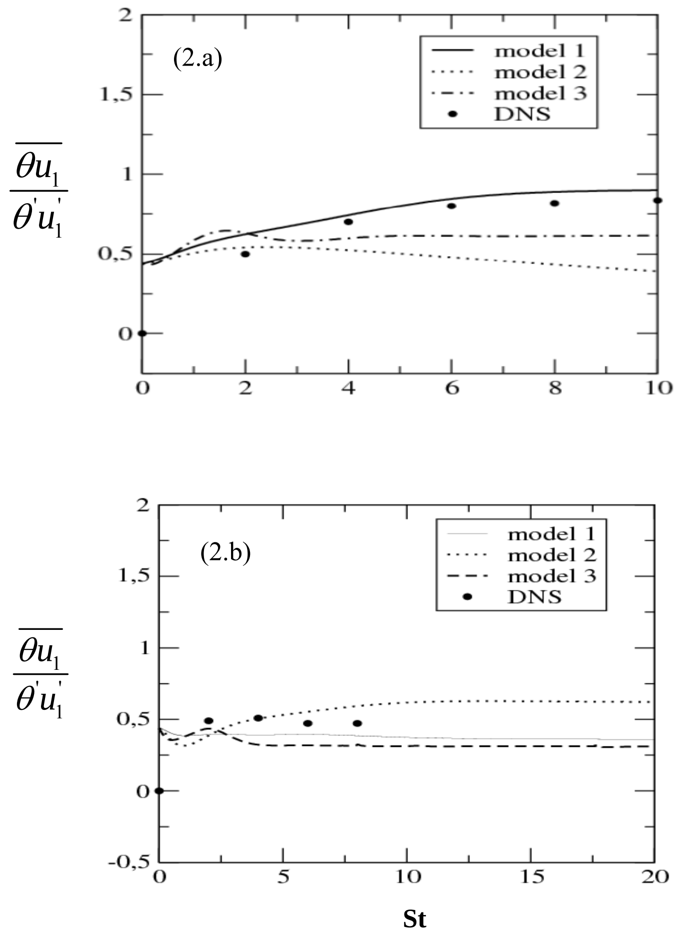

Figure 2 : time evolutions of the correlation coefficient for the three models in the cases R=0 (2.a) and R=0.25 (2.b)

On figures (3.a) and (3.b), are plotted time evolution of thermal ratio $\rho_3 = \dfrac{\theta'/G}{q'/S}$. In the case R=0, agreement is observed between model 1 and DNS results for $St\rangle 5$ but between model 2 and DNS results for $4\langle St\langle 6$.

Equilibrium asymptotic behavior is clearly observed according to the models 1 and 3 on the time evolution of ρ_3.

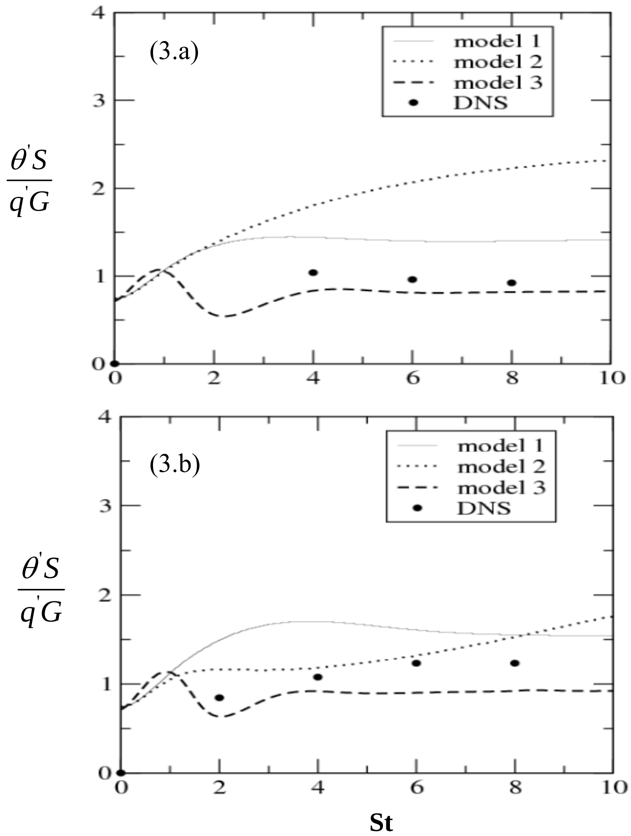

Figure 3 : time evolutions of the ratio $\frac{\theta' S}{q' G}$ for the three models in the cases R=0 (3.a) and R=0.25 (3.b)

5. Conclusions

In this work, thermal field of homogeneous sheared turbulence in a rotating frame have been studied. Second order modeling have been carried out in order to closure a set of three nonlinear systems of equations obtained from retaining three common second order closure models that are called model 1, model 2 and model 3. After that, we are opting for numerical integration which is conducted by the fourth Runge Kutta method for various values of rotating number R, in order to analyze the prediction of equilibrium asymptotic behavior of dimensionless parameters describing thermal rotating turbulence.

By means of this numerical study, equilibrium asymptotic states have been successfully confirmed through second order models retained here where the LRR and SL models have shown the most strong agreement with DNS results of Brethouwer. In addition, we have shown a considerable influence of rotation both on the time evolution of thermal parameters and on the equilibrium asymptotic values.

Nomenclature

Symbol	Name	Unity	Symbol	Name	Unity
S	mean shear rate	s^{-1}	$b_{ij} = \overline{u_i u_j}/2k - \delta_{ij}/3$	Reynolds anisotropic tensor	
G	mean temperature gradient	$°Cm^{-1}$	$k = \overline{u_i u_i}/2$	Kinetic energy	$m^2 s^{-2}$
$R=2\Omega/S$	Dimensionless rotation number		$\overline{U}_{p,q}$	Velocity gradient	s^{-1}

References

[1] S.V Poroseva, S.C Kassinos, C.A Langer and W.C Reynolds, Structure-based turbulence model : application to a rotating pipe low, Phys.Fluids, 14,1523-1532, 2002.

[2] C.G Speziale and N.M.G Mhuris, On the prediction of equilibrium states in homogeneous turbulence, J.Fluid Mech, 209, 591-615, 1989.

[3] B.E Launder, G Reece and W Rodi, Progress in the development of a Reynolds stress closure, J.Fluid.Mech, 68, 537-572, 1975.

[4] C.G Speziale and N.M.G Mhuris, Calling laws for homogeneous turbulent shear flows in a rotating frame, NASA Langley research center, Hampton, Virginia 23665, 1988.

[5] T. Shih, J. Chen and L. Lumley, Second order modeling of boundary-free turbulent shear flows with a new model form of pressure correlation, Sibley School of Mech. And Aerospace Eng, Comell University, Rept. FDA-85-3, 1985a.

[6] G. Brethouwer, The effect of rotation on rapidly sheared homogeneous turbulence and passive scalar transport, Linear theory and direct numerical simulation, J.Fluid Mech, 542, 305-342, 2005.

[7] A. Cadiou, Contribution à l'étude de modèles de turbulence au second ordre, Thèse de doctorat, École centrale de Nantes, 1996.

[8] T. Shih, J. Chen and L. Lumley, Modeling of pressure correlation terms in Reynolds Stress and Scalar flux equations, Sibley School of Mech. And Aerospace Eng, Comell University, Rept. FDA-85-3, 1985b.

[9] T. Shih, T.H, Constitutives relations and realizability of single-point turbulence closures in turbulence transition and modeling, Edited by D.S. Hallback, Hennigson, A.V. Johansson and P.H. Alfredsson, Dordnech, 1996.

[10] S. Tavoularis and S. Corrsin, Experiments in nearly homogeneous turbulent shear flow with a uniform mean temperature gradient, Part1. J. Fluid Mech, 104, 311-347, 1981.