18èmes Journées Internationales de Thermique (JITH 2017)
Monastir (Tunisie), 25 - 27 Octobre 2017

Conjugate natural convection in a porous enclosure

A. Belazizia[1, 2, 3], S. Benissaad[2] and S. Abboudi[3]

[1] Université Larbi Tebessi Tebessa Algérie.
[2] Laboratoire d'Energétique Appliquée et de la Pollution. Université Mentouri Constantine1. Algérie.
[3] Institut IRTES, Laboratoire M3M, site de Sévenans, UTBM, 90000 Belfort Cedex, France
Phone : 33(3)84583036, Email : nacer.belazizia@gmail.com

ABSTRACT. *Steady, laminar, conjugate natural convection flow in a square porous enclosure is considered. The enclosure is filled with air and subjected to horizontal temperature gradient. Darcy-Brinkman-Forchheimer model is considered. Finite volume method is used to solve the dimensionless governing equations. The main focus of the study is on examining the effect of Darcy number on fluid flow and heat transfer. The effect of Rayleigh number and conduction in the left wall is also considered. The obtained results show that natural convection can be strength by the increase of both Rayleigh number and conductivity ratio, because of the increase of the effective temperature difference driving the flow. It is also intensified as the Darcy number increases. Heat transfer is less important for low permeability of porous medium. Furthermore for poor conducting wall, where the solid part is an insulated material and the thermal resistance is more important the average Nusselt number is approximately constant and having low values comparing with equal and high conducting wall, indicating that most of heat transfer is by heat conduction.*
Keywords: Conjugate natural convection, porous medium, finite volume method, Darcy-Brinkman- Forchheimer model.

1. INTRODUCTION

Natural convection heat transfer in a cavity filled with fluid saturated porous media can be seen in many applications of engineering. Some of these are solar power collectors, geothermal applications, nuclear reactors cryogenic storage, furnace design and others [5]. The problem of natural convection flow in a square and rectangular enclosure with uniform temperature at vertical walls and insulated top and bottom walls has been the subject of many studies. The walls of the enclosure are assumed to be of zero thickness and conduction is not accounted for. However, in many practical situations, especially those concerned with the design of thermal insulation, conduction in the walls can have an important effect on the natural convection flow in the enclosure [2-4]. Natural convection in porous medium is studied in many articles [5-7]. Yasin and al.[5] analyzed numerically using finite difference scheme and Darcy model the flow and heat

transfer in a diagonally divided square cavity by an inclined plate and filled with a porous medium. Vertical walls are kept at isothermal conditions, while horizontal walls are insulated.
Sathiyamoorthy and al [6] studied numerically natural convection flows in a square cavity filled with a porous matrix. Darcy–Forchheimer model without the inertia term is used to simulate the momentum transfer in the porous medium.
Tanmay and al.[7] studied numerically natural convection flows in a square cavity filled with a porous matrix using penalty finite element method for uniformly and non-uniformly heated bottom wall, and adiabatic top wall maintaining constant temperature of cold vertical walls. Darcy–Forchheimer model is used to simulate the momentum transfer in the porous medium.
The above results concern either pure convection in porous medium without using Darcy Brinkman forchheimer model. It is essential to study conjugate natural convection using the general model. We study the effect of the permeability of porous medium and wall conductivity on steady laminar conjugate natural convection in a square enclosure filled with air (Pr=0.71) and submitted to horizontal temperature gradient. The main focus is on examining the effect of conduction in the wall, Rayleigh number and Darcy number on fluid flow and heat transfer.

2. PROBLEM GEOMETRY AND GOVERNING EQUATIONS

The geometry of the problem is shown in Fig.1. The flow is two-dimensional, laminar and incompressible. All fluid properties are constant. The fluid is considered to be Newtonian. Viscous dissipation, heat generation and radiation effects are neglected. The Boussinesq approximation is applied : $\rho(T)=\rho_0[1-\beta(T-T_0)]$.

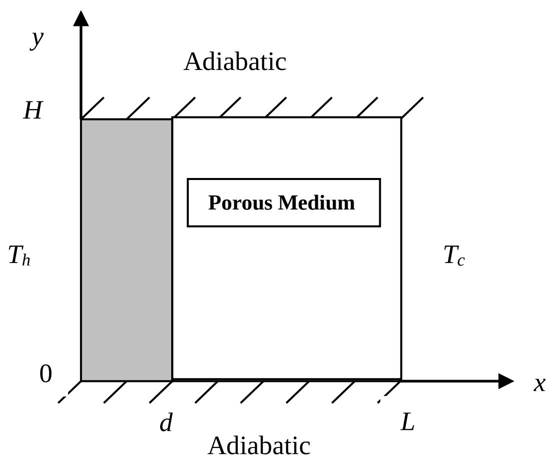


Figure 1. Physical configuration.

The dimensionless form of the governing equations can be written as:

- at $t=0$; $U=V=\theta_w=\theta_f=0$
- For $t>0$

- ***fluid part***

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$\frac{1}{\varepsilon}\left(\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}\right)=-\frac{1}{Da}\frac{\partial P}{\partial X}+\frac{1}{\varepsilon}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)-\left(\frac{1}{Da}+\frac{CI}{\sqrt{Da}}\left|\vec{V}\right|\right)U \tag{2}$$

$$\frac{1}{\varepsilon}\left(\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right)=-\frac{1}{Da}\frac{\partial P}{\partial Y}+\frac{1}{\varepsilon}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra\theta-\left(\frac{1}{Da}+\frac{CI}{\sqrt{Da}}\left|\vec{V}\right|\right)V \tag{3}$$

$$\frac{\partial \theta_f}{\partial t}+U\frac{\partial \theta_f}{\partial X}+V\frac{\partial \theta_f}{\partial Y}=\left(\frac{\partial^2 \theta_f}{\partial X^2}+\frac{\partial^2 \theta_f}{\partial Y^2}\right) \tag{4}$$

● ***solid part***

$$\frac{\partial \theta_w}{\partial t}=\alpha^*\left(\frac{\partial^2 \theta_w}{\partial X^2}+\frac{\partial^2 \theta_w}{\partial Y^2}\right) \tag{5}$$

The effect of porous medium appears in the governing equations (2-3) through Darcy and porosity numbers.
The boundary conditions are:

$$U(0,Y)=U(L/H,Y)=U(X,0)=U(X,1)=0\ ;\ V(0,Y)=V(L/H,Y)=V(X,0)=V(X,1)=0 \tag{6a}$$

$$\theta_w(0,Y)=1;\ \theta_f(L/H,Y)=0;\ \frac{\partial \theta_w(X,0)}{\partial Y}=0;\frac{\partial \theta_f(X,0)}{\partial Y}=0;\frac{\partial \theta_w(X,1)}{\partial Y}=0\ ;\frac{\partial \theta_f(X,1)}{\partial Y}=0 \tag{6b}$$

$$\theta_f(D,Y)=\theta_w(D,Y);\ \left.\frac{\partial \theta_f(D,Y)}{\partial X}\right|_{fluide}=Kr\left.\frac{\partial \theta_w(D,Y)}{\partial X}\right|_{solide} \tag{6c}$$

The local and average Nusselt numbers are defined by:

$$Nu=\left(-\frac{\partial \theta}{\partial X}\right)_{X=D,L/H}\ ;\ \ \overline{Nu}=\int_0^1 Nu\, dY \tag{7}$$

A hybrid scheme and first order implicit temporally discretisation are used [1]. The iteration process is terminated under the following conditions:

$$\sum_{i,j}\left|\phi_{i,j}^{n}-\phi_{i,j}^{n-1}\right|/\sum_{i,j}\left|\phi_{i,j}^{n}\right|\le 10^{-5} \tag{8}$$

where ϕ representes *U*, *V* and *θ*.

$$\text{For wall side}\quad \overline{Nu}\Big|_{X=0}=\overline{Nu}\Big|_{X=D} \tag{9}$$

$$\text{For fluid side}\quad \overline{Nu}\Big|_{X=D}=\overline{Nu}\Big|_{X=L/H} \tag{10}$$

Where *D* is the dimensionless wall thickness (*D=d/H).*

3. GRID INDEPENDENCY

Figure 2 shows, for *D=0.2*, $Ra=10^6$, Kr=1and $Da=10^{-3}$ the effect of grid points on the average Nusselt number at solid-porous interface. In order to obtain a precise results a (90x90) grid points was selected and used in all the computations.

4. NUMERICAL VALIDATION

In order to validate our results, a comparison with the results obtained by Kaminski and Prakash (1986) for conjugate natural convection in fluid medium was made (table 1). A good agreement between the obtained and reported results can be observed.

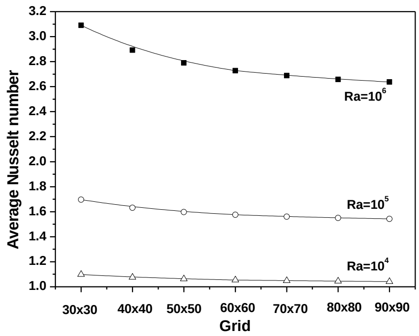


Figure 2. Average Nusselt Number for different grid size.

Table 1 Comparison with Kaminski solution, (*D=0.2*).

Ra	*Kr*	Kaminski	Present Study
7×10^2	1	0.87	0.88
	5	1.02	1.02
	10	1.04	1.04
	∞	1.06	1.06
7×10^4	1	2.08	2.16
	5	3.42	3.45
	10	3.72	3.74
	∞	4.08	4.06

5. RESULTS AND DISCUSSION

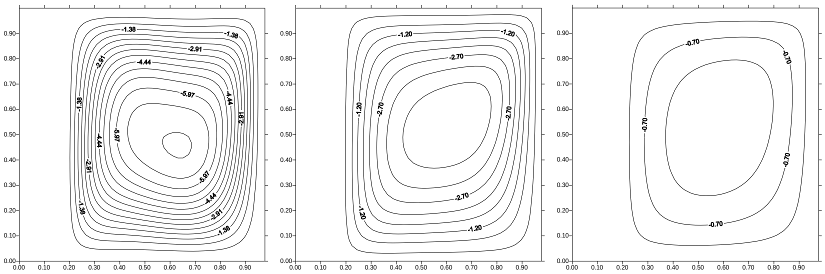
The results are presented for different values of governing parameters: $10^3 \leq Ra \leq 10^6$, $10^{-4} \leq Da \leq 10^{-1}$, $0.1 \leq Kr \leq 10$ and D=0.2.

5.1 Fluid Motion and Thermal field

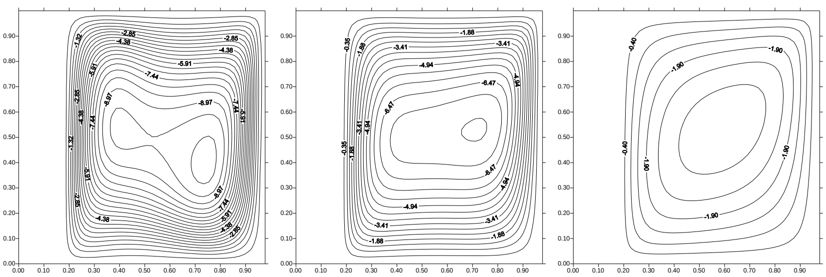
Table 2 and figure3 (a-c) show the effect of both Darcy number and thermal conductivity ratio on fluid motion in the enclosure for poorly conducting wall (*Kr*=0.1), equal wall/porous conductivity (*Kr*=1) and high conducting wall (*Kr*=10). We observe in table 1 that the maximum values of dimensionless stream function $/\psi_{max}/$ and fluid velocity V_{max} in porous part are decreasing with the decrease of Darcy number and increasing with the increase of conductivity ratio. As a result natural convection is weaken because of the decrease of porous permeability and strength because of the increase of the effective temperature difference driving the flow.

(a) Kr=0.1

$Da=10^{-1}$ $Da=10^{-3}$ $Da=10^{-4}$



(b) Kr=1



(c) Kr=10

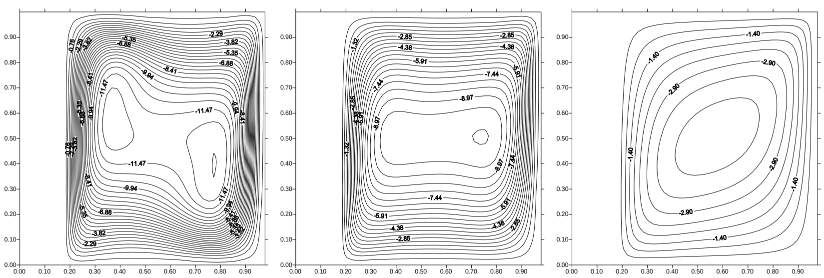


Figure 3. Steady state of stream lines for different values of *Da* and *Kr*.

Table 2. $/\psi_{max}/$ and V_{max} for differante *Da* and *Kr*. *D*=0.2

	$Da=10^{-1}$		$Da=10^{-3}$		$Da=10^{-4}$	
	$/\psi_{max}/$	V_{max}	$/\psi_{max}/$	V_{max}	$/\psi_{max}/$	V_{max}
Kr=0.1	7.062	44.183	4.145	28.059	1.475	8.329
Kr=1	10.205	90.656	7.549	68.422	3.232	23.659
Kr=10	12.515	127.646	10.027	100.221	4.218	32.450

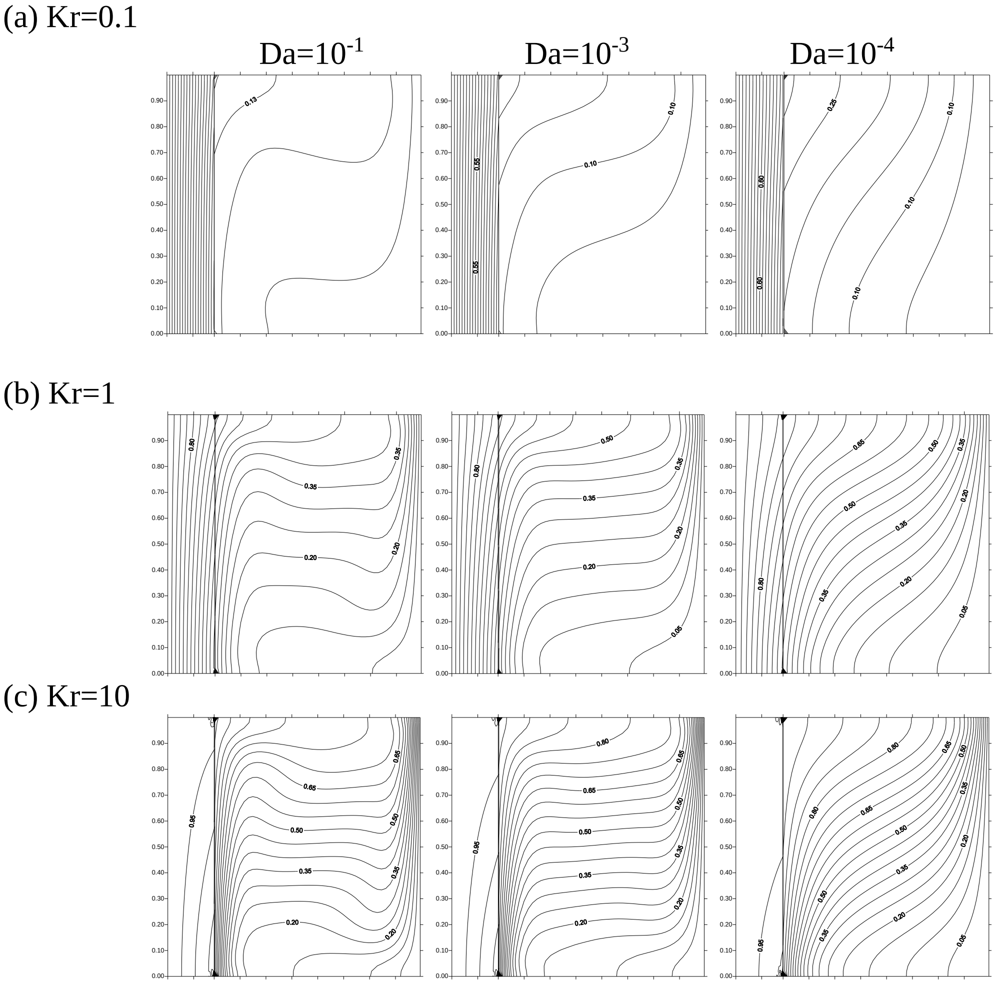


Figure 4. Steady state of isotherms for different values of *Da* and *Kr*.

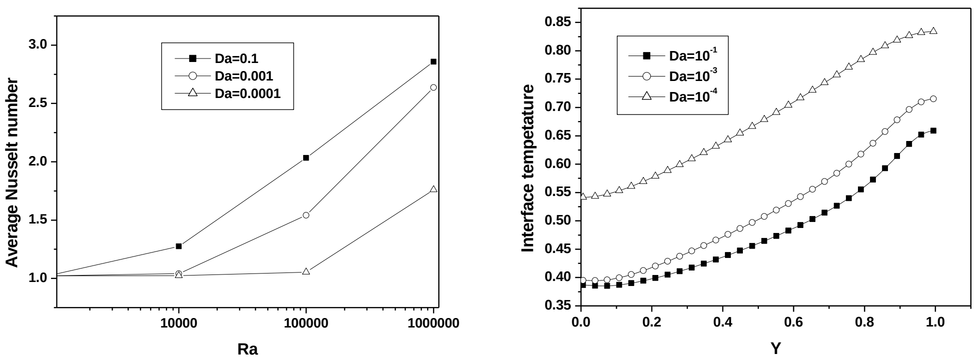
The fluid rises along the hot wall/porous surface and falls along the right cold wall, so thermal gradient is very important in these regions. For low *Da* and *Kr* the isotherms shown in Fig.4 (a) are almost parallel to the vertical walls, indicating that most of heat transfer is by heat conduction. As *Da* and *Kr* increase convection in porous part is initiated and temperature lines change their shape. For high *Da* and *Kr* ($Da=10^{-1}$, $Kr=10$), we observe in Fig.4(c) a temperature stratification in the vertical direction and the thermal boundary layer is well established along the side walls, as a consequence the development of convection mode of heat transfer.

For poor conductive wall ($Kr=0.1$) where the solid part is an insulation material, the average Nusselt number is almost constant and having low values compared with those ($Kr=1$ and 10). This is a logical result since reducing the thermal conducting of the wall leads to the increase in thermal resistance of the overall system and therefore reducing $\overline{Nu}$. This indicates that most of heat transfer is dominated by conduction mode. In addition for high conduction wall ($Kr=10$) where the solid part is a good conductive wall convection heat transfer is strength and the solid layer tends to become an isothermal wall.

5.2 Average Nusselt Number and Interface Temperature

The rate of heat transfer across the cavity is obtained by evaluating the average Nusselt number at the active side walls. It is clear from the figure 5(a) that the average Nusselt number is more important for high Darcy numbers. It is increasing with the increase of Rayleigh number for a given *Da*. This means that convection heat transfer becomes more important for both high *Ra* and *Da*.

Figure 5(b) shows for *Kr=1* and *Ra*=10^6 the effect of Darcy number on wall/porous interface temperature. The temperature difference between the interface and the cold boundary ($\theta = 0$) is small for porous medium with high permeability. It becomes more important with the decrease of Da, and leads to increase the average Nusselt number. The temperature profile across the wall/ porous interface is quite non uniform. This non uniformity has a noticeable effect on the flow field and the flow structure is asymmetric.



(a) **(b)**

Figure 5. **(a)** Variation of $\overline{Nu}$ with Rayleigh number Ra. **(b)** Variation of wall/porous interface temperature $D = 0.2$, Kr=1.

6. CONCLUSION

A numerical study of conjugate natural convection in porous medium was employed to analyze the flow and heat transfer of air filled in a square enclosure with thick and conductive left vertical wall. The following conclusions were summarized. Natural convection is strength by the increase of both Rayleigh number and conductivity ratio because of the increase of the effective temperature difference driving the flow. Flow velocity and thermal field are higher when the permeability medium is more important. The interface temperature is found to be quite non- uniform. This non uniformity tends to make the flow pattern in the enclosure asymmetric. Furthermore for poor conducting wall (*Kr*=0.1) and less Darcy number convection flow is dominated by heat conduction for both wall and porous layer.

NOMENCLATURE

D dimensionless wall thickness *D=d/H*

Da	Darcy Number K/H^2	
H	wall height	m
K	porous permeability.	
Kr	thermal conductivity ratio $Kr=k_w/k_f$	
k_w	thermal conductivity of the solid wall	$wm^{-1}k^{-1}$
k_f	thermal conductivity of the fluid	$wm^{-1}k^{-1}$
L	cavity length	m
Nu	local Nusselt number,	
$\overline{Nu}$	average Nusselt number,	
Pr	Prandtl number of the fluid (υ/α)	
Ra	Rayleigh number $g\beta H^3(T_h-T_c)/\upsilon\alpha$	
T	temperature	K
t^*	time	s
t	dimensionless time $t^*/(H^2/\alpha)$	
u, v	velocity components	ms^{-1}
U, V	dimensionless velocity components $u/(\alpha/H)$, $v/(\alpha/H)$	
$\mathbf{V}$	dimensionless velocity of the flow	
p	pressure	$N.m^{-2}$
P	dimensionless pressure $p/(\alpha/H)^2$	
X, Y	non-dimensional Cartesian coordinates, x/H, y/H	

Greek symbols

α^*	thermal diffusivity ratio	α_w / α_f
β	thermal expansion coefficient	k^{-1}
θ	non-dimensional temperature $(T-T_c)/(T_h-T_c)$	
υ	kinematics viscosity	m^2s^{-1}
ψ	non-dimensional stream function $U = \partial\psi/\partial Y$	
ϕ	independent variable (ϕ =U, V or θ)	
ε	porosity of porous medium	

Subscripts

c	cold
f	fluid
h	hot
w	wall
wf	wall/fluid interface

REFERENCES

1. Patankar, S.V. (1980) Numerical Heat Transfer and Fluid Flow. *Hemisphere, McGraw-Hill,Washington, DC.*
2. Kaminski, D.A., Prakash, C. (1986) Conjugate natural convection in a square enclosure: effect of conduction in one of the vertical walls, *Int. J. Heat Mass transfer* 12, 1979-1988.
3. Kim, D.M., Viskanta, R. (1985) Effect of wall heat conduction on natural convection heat transfer in a square enclosure, *J. Heat Transfer* 107, 139-146.

4. Liaqat, A., Baytas, A.C. (2001) Conjugate natural convection in a square enclosure containing volumetric sources, *Int. J. Heat Mass transfer*. 44, 3273-3280.
5. Yasin Varol, Hakan. Oztop, Ioan Pop, Natural convection in a diagonally divided square cavity filled with a porous medium ,Int. J. of Thermal Sciences 48 (2009) 1405–1415
6. Sathiyamoorthy, M., Tanmay Basak, Roy, Pop, I., Steady natural convection flow in a square cavity filled with a porous medium for linearly heated side wall(s), *Int. J. Heat and Mass Transfer* 50 (2007) 1892–1901
7. Tanmay Basak, S., Roy, T., Paul, I. Pop, Natural convection in a square cavity filled with a porous medium effects of various thermal boundary conditions , Int. *J. Heat Mass Transfer* 49 (2006) 1430–1441