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Influence of viscous dissipation on irreversibility in a saturated porous cavity under the Darcy-Brinkman formulation

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Abstract : This work deals with the influence of the viscous dissipation on the mixed convection heat transfer in a square saturated porous cavity filled with an incompressible, Newtonian fluid. Three models are investigated the first is related to the Darcy, the second represents the Darcy- Nield model and the third is for Darcy and El Hadhrami. The vertical walls of the cavity are subject to thermal temperature gradient. A numerical program written in COMSOL Multiphysics software, was developed, to solve the Navier Stokes and energy equations, under the Darcy–Brinkman formulation. This investigation is principally focalized on the contribution of each term of the three models cited above on irreversibility, heat transfer and flow structure. The influence of other governing parameters, on thermodynamics' irreversibility, is also studied.

Key words: Viscous dissipation. Mixed convection .Porous media . Darcy- Nieled. Darcy-Elhadhrami . Entropy Generation .

Introduction:

Entropy generation is directly related with the thermodynamic irreversibility because it encountered all heat transfer process. The viscous dissipation are important in geophysical flows, polymer processing and also in certain industrial processes. Many literature concerning convective flow in a porous media is abundant in the recent books by Nield and Bejan [1] ,Vafai [2], and Pop and Ingham [3].

Ingham et al.[4] analyzed the effect of viscous dissipation for the cases of symmetric and asymmetrically heated between two vertical walls filled with a porous media in free and forced convection. They noted that when the viscous dissipation is neglected the solution is singular at critical Rayleigh numbers, and when it taken into consideration, solutions exist at all Rayleigh numbers. Al-Hadhrami et al.[5] investigate the effect of viscous dissipation For the case of wall temperature decreasing linearly with height, they noted that for any value of the Rayleigh number there were two solutions mathematically, but only one of them is physically acceptable. Barletta [6] analyzes the laminar mixed convection in a plane vertical channel by taking into account the viscous dissipation. focus into dimensionless velocity ,dimensionless temperature and the Nusselt number have been evaluated both in the case of asymmetric and symmetric heating has been shown that the effect of viscous dissipation can be importat in the case of upward flow.
Basak et al. [7] presented the natural convection flows in a square cavity filled with a porous medium matrix has been studied numerically using the effects of various thermal boundary conditions. The results

display the circulations and temperature distributions within the cavity and the heat transfer rate at the heated wall in terms of local and average Nusselt numbers.

The prime objective of this study is to consider the effect of different viscous dissipation terms in the square cavity filled with porous media. In this work the Renolds, the Rayleigh, the Brinkman numbers are fixed respectively and the Prandlt are varied 0.3 , 0.7 and 1

1. Mathematical Modeling:

Consider the mixed convection flow in a lid-driven cavity filled with Newtonian incompressible saturated porous medium. There is a temperature difference between the right and left vertical isothermal walls while the top and the bottom are well insulated. The cavity walls are assumed rigid and impermeable.

The left vertical wall is assumed at the constant high temperature of Th while the right vertical wall is at low temperature of Tc. As seen in the figure (1) .The fluid is assumed to be incompressible.

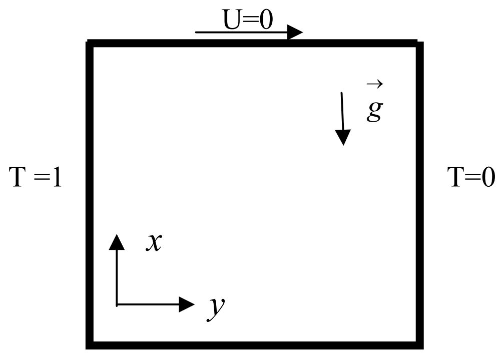


Fig. 1 Mathematical model

Boundary conditions are:

For all walls: u=v=0

For X=0, 0≤Y≤ 1, U=V=0, T=1 / For X=1, 0≤Y≤ 1, U=V=0, T=0/ For Y=0, 0≤X≤ 1, U=V=0, $\frac{\partial T}{\partial Y}=0$

For Y=1, 0≤Y≤ 1, U=1, V=0, $\frac{\partial T}{\partial Y}=0$

The fluid density satisfies the Boussinesq approximation:

$$\rho = \rho_0\left[1-\beta_T(T-T_0)\right] \quad (1)$$

In the equation above, ρ_0 , T_0 and β_T are the fluid density, the reference temperature, and the thermal volumetric expansion coefficient, respectively. The latter is given by:

$$\beta_T = \frac{1}{\rho_0}\left(\frac{\partial p}{\partial T}\right)_P \quad (2)$$

Using Darcy-Brinkman formulation and in two-coordinate system, the governing dimensional equations related to this flow:

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}=0 \tag{3}$$

$$\rho_0\left[\frac{1}{\varepsilon}\frac{\partial v_y}{\partial t}+\frac{1}{\varepsilon^2}v_x\frac{\partial v_y}{\partial x}+\frac{1}{\varepsilon^2}v_y\frac{\partial v_y}{\partial y}\right]=-\frac{\partial p}{\partial y}-\frac{\mu}{K}v_y+\mu_{eff}\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}\right)-\rho g \tag{4}$$

$$\rho_0\left[\frac{1}{\varepsilon}\frac{\partial v_x}{\partial t}+\frac{1}{\varepsilon^2}v_x\frac{\partial v_x}{\partial x}+\frac{1}{\varepsilon^2}v_y\frac{\partial v_x}{\partial y}\right]=-\frac{\partial p}{\partial x}-\frac{\mu}{K}v_x+\mu_{eff}\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}\right) \tag{5}$$

$$\sigma\frac{\partial T}{\partial t}+\left[v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}\right]=\alpha_{eff}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{\Phi}{(\rho c)_\rho} \tag{6}$$

Were σ specific heat capacities ratio, μ_{eff} is the effective viscosity, μ is the fluid dynamic viscosity, K is the permeability and ε is the medium porosity.

The term Φ is the viscous dissipation which appears as an internal heat source in the porous media, which was defined by:

$$\Phi=\frac{u}{k}\sum_i V_i^2-C_1u_{eff}\sum V_i\sum_j\left(\frac{\partial^2 V_i}{\partial x_j^2}\right)+C_2\mu\sum\left(\frac{\partial V_i}{\partial x_j}\right)\left(\frac{\partial V_i}{\partial x_j}+\frac{\partial V_i}{\partial x_i}\right) \tag{7}$$

$$\Phi=\frac{u}{K}(v_x^2+v_y^2)-C_1\mu_{eff}\left[v_x\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}\right)+v_y\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}\right)\right]$$
$$+C_2\mu\left[2\left(\frac{\partial v_x}{\partial x}\right)^2+2\left(\frac{\partial v_y}{\partial y}\right)^2+\left(\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial x}\right)^2\right] \tag{8}$$

Four models will be investigated :

-Model 1is related to Darcy (C1=C2=0) ,

-Model 2 presented the Nield model (C1=1, C2=0) [8] ,

-Model 3obtainable El-Hadhrami model **(**C1=0, C2=1) [9] ,

-Model 4 were (C1=1, C2=1).

The specific heat capacity ratio and the effective thermal diffusivity are defined by:

$$\alpha_{eff}=\frac{k_m}{(\rho c)_\rho}\quad \sigma=\frac{(\rho c)_m}{(\rho c)_\rho}$$

Where $(\rho c)_m$ and $(\rho c)_\rho$ are the specific heat capacity per unit volume of the porous medium and the specific heat capacity of the fluid, respectively.

The effective thermal conductivity of a porous medium K_m can be written as the weighed arithmetic mean of the solid phase and the fluid phase conductivities:

$$k_m = (1-\varepsilon)k_s + \varepsilon k_f \tag{9}$$

With k_f are the thermal conductivities of fluid and k_s the thermal conductivities of solid.

The dimensionless variables are:

$$X = \frac{x}{H}, Y = \frac{y}{H}, V_x = \frac{v_x}{u_0}, V_y = \frac{v_y}{u_0}, \theta = \frac{T - T_0}{\Delta T}, \Pr = \frac{\mu Cp}{K_m}, \mathrm{Re} = \frac{Hu_0}{u},$$
$$Da = \frac{k}{H^2}, Br = \frac{\mu u_0}{k_m \Delta T}, Ec = \frac{u_0^2}{Cp\Delta T}, \Lambda = \frac{\mu_{eff}}{\mu}, Ra = \frac{\beta g \Delta T H^3}{\mu_{eff}} \tag{10}$$

Using the dimensionless variables mentioned above, the governing equations can be written in dimensionless form as:

$$div(V) = 0 \tag{11}$$

$$\frac{\partial V_x}{\partial \tau} + div(J_{V_x}) = -\varepsilon \frac{\partial P}{\partial X} - \frac{\varepsilon}{Da.\mathrm{Re}} V_x \tag{12}$$

$$\frac{\partial V_y}{\partial \tau} + div(J_{V_y}) = -\varepsilon \frac{\partial P}{\partial Y} - \frac{\varepsilon}{Da.\mathrm{Re}} V_y + \frac{Ra.\varepsilon}{\mathrm{Re}.Pe}\theta \tag{13}$$

$$\sigma\frac{\partial \theta}{\partial \tau} + div(J_\theta) = \frac{Br}{\mathrm{Re}.\Pr.Da}(V_x^2 + V_y^2) + c_1\Lambda\frac{Br}{\mathrm{Re}\Pr}\left[V_x(\frac{\partial^2 V_x}{\partial X^2} + \frac{\partial^2 V_x}{\partial Y^2}) + V_y(\frac{\partial^2 V_y}{\partial X^2} + \frac{\partial^2 V_y}{\partial Y^2})\right] + c_2\frac{Br}{\mathrm{Re}\Pr}\left[2(\frac{\partial V_x}{\partial X})^2 + 2(\frac{\partial V_y}{\partial Y})^2 + (\frac{\partial V_x}{\partial Y} + \frac{\partial V_y}{\partial X})^2\right] \tag{14}$$

Where:

$$\left.\begin{aligned} J_{V_x} &= \frac{1}{\varepsilon}V_x.V - \frac{\Lambda\varepsilon}{\mathrm{Re}} grad(V_x) \\ J_{V_y} &= \frac{1}{\varepsilon}V_y.V - \frac{\Lambda\varepsilon}{\mathrm{Re}} grad(V_y) \\ J_\theta &= \theta V - \frac{1}{\mathrm{Re}.\Pr} grad(\theta) \end{aligned}\right\} \tag{15}$$

2. Entropy production:

For the flow in porous medium, the local entropy generation can be written as follows (10)

$$S = \frac{k_m}{T_0^2}\left[(\frac{\partial T}{\partial x})^2 + (\frac{\partial T}{\partial y})^2\right] + \frac{\mu}{T_0.K}(v_x^2 + v_y^2) + \frac{\mu}{T_0}\left[2(\frac{\partial v_x}{\partial x})^2 + 2(\frac{\partial v_y}{\partial y})^2 + (\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x})^2\right] \tag{16}$$

The first term of the entropy generation equation is the heat transfer irreversibility, the second relied to the Darcy viscous irreversibility and the third represent the clear fluid viscous irreversibility.

$$Si_l = Si_{Th} + Si_D + Si_F$$

Using the dimensionless variable mentioned in equation (10) the governing equation can be written in dimensionless form:

$$Si_{Th} = \left(\frac{\partial \theta}{\partial X}\right)^2 + \left(\frac{\partial \theta}{\partial Y}\right)^2 \tag{17}$$

$$Si_D = \frac{Br^*}{Da}(V_x^2 + V_y^2) \tag{18}$$

$$Si_F = Br^*\left[2\left(\frac{\partial V_x}{\partial X}\right)^2 + 2\left(\frac{\partial V_Y}{\partial Y}\right)^2 + \left(\frac{\partial V_X}{\partial Y} + \frac{\partial V_Y}{\partial X}\right)^2\right] \tag{19}$$

Where Br^* is defined as the modified Brinkman number, it's given by:

$$Br^* = \frac{Br}{\Omega}$$

Br and Ω are the Brinkman number and the dimensionless temperature difference, respectively. They are given by:

$$Br = \frac{u_0^2 \mu}{k_m . \Delta T} \quad \Omega = \frac{\Delta T}{T_0}$$

The dimensionless total entropy generation for the entire channel is obtained by integrating

$$S_{tot} = \int_V Si_l \, dv$$

The Bejan number compares the magnitude of entropy generation due to heat transfer with the magnitude of the total entropy generation when:

$$Be = \frac{S_{thermal}}{S_{Total}}$$

$Be \leq 1$, The irreversibility contributed to heat transfer dominates

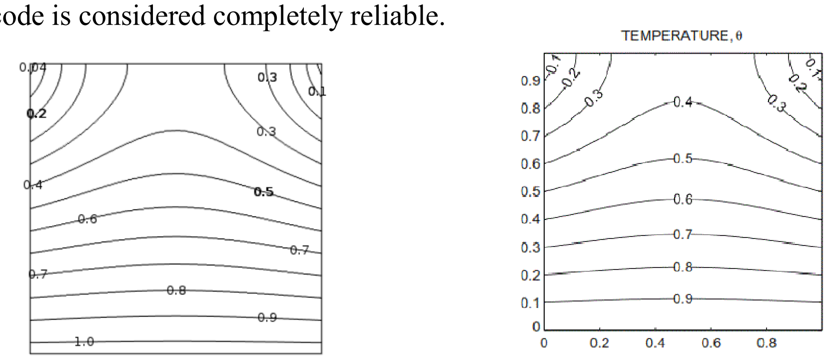
$Be \geq 1$, The irreversibility contributed to viscous effects dominates

3. Numerical method

We solve the model of Navier Stokes and energy equation in non-dimensional form for an incompressible Newtonian fluid, consisting on the equations of conservation of momentum, with the appropriate boundary conditions. COMSOL Multiphysics bases the discretization of the equations on the finite element method. No slip boundary condition is used on the walls of the cavity.

In order to validate our results, the solutions given b the present code are compared with those obtained by Basak et al.[7] for Pr=0.015, Re=1, Da= 10^{-4}, Gr=10^{5}, Pr=10

We obtain the same parameter of Bask et al [7] with the same boundary conditions and we neglected the effect of the viscous dissipation. The results of this comparison are illustrated by figure (2) in terms of streamline and isotherms. From these results it is evident that the present code generates results are in good agreement with those of Basak et al [7]. Hence the present code is considered completely reliable.



Present code (a) Basak et al. [7] code (b)

Figure (2) : Validation

4. Result and discussion.

Given the large number of variables related to the following study, the results are reported for a typical case of Porosity, Rayleigh, Brinkman, and Reynolds number are fixed at 0.8, 10^{5} ,10^{-4}, 20 respectively and variation of Prandlt 0.3, 0.7 and 1

Figs 1,2,3 and 4 show the effect of viscous dissipation for the entropy generation according to different models.

4.1. Total entropy generation:

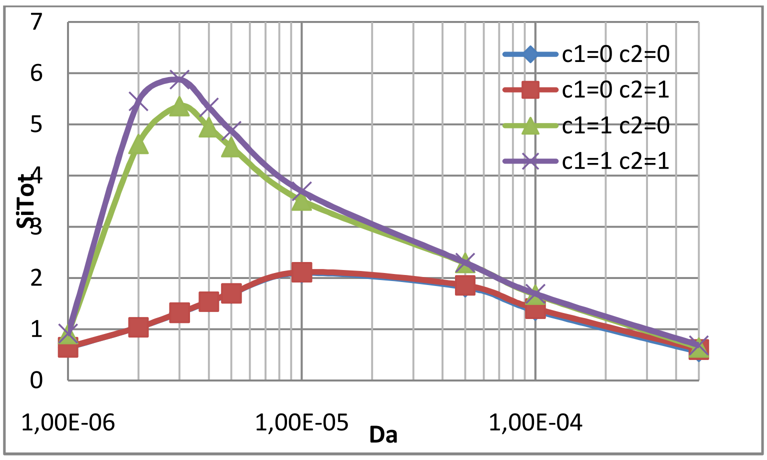


Fig.1. Total entropy generation as function as Darcy number

Fig.1 evince that the total entropy generation for Darcy less than 210^{-6} increase for the two models 1 and 3 but for the models 2 and 4 the total entropy generation increase slowly to attempt Da 10^{-5} .For Darcy great than 10^{-2}, the total entropy generation decrease progressively for models 1 and 3 , and the decrease of model 2 and 4 after Da 1 10^{-5}.

From this figure we conclude that the effect of the two couples related to the four viscous dissipation models is appreciable when Darcy number less than $10^{-5.}$ While when Darcy number is greater than 5 10^{-5} , the four models are practically identical and one can observe a decrease on the total irreversibility as the Darcy number increases.

The model 3 and 4 have a persist effect for total entropy generation so we can conclude that the effect of Nield term in viscous dissipation equation dominate .

4.2. Darcy number effect for Bejan number in model 1

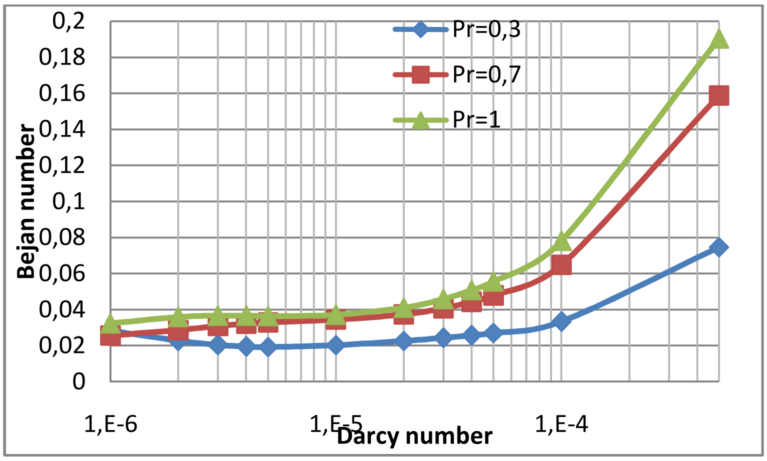


Figure 2. Variation of Bejan number versus Darcy number for different Prandlt in model 1

Fig.2 show that the variation of Bejan for different Prandlt 0.3 , 0.7 and 1 number are the same behaviors .For Darcy less than 10^{-4} it's remarkable that at very low Darcy the variation of Bejan number for three Prandlt take approachably the same value who's tends to zero and it change with slow variation while when Da more than 10^{-4} the different variation increase progressively with the increasing of Darcy number. in this case the viscous energy received in the system due essentially to the viscous dissipation in the cavity.

4.3. Darcy number effect for Bejan number in model 2

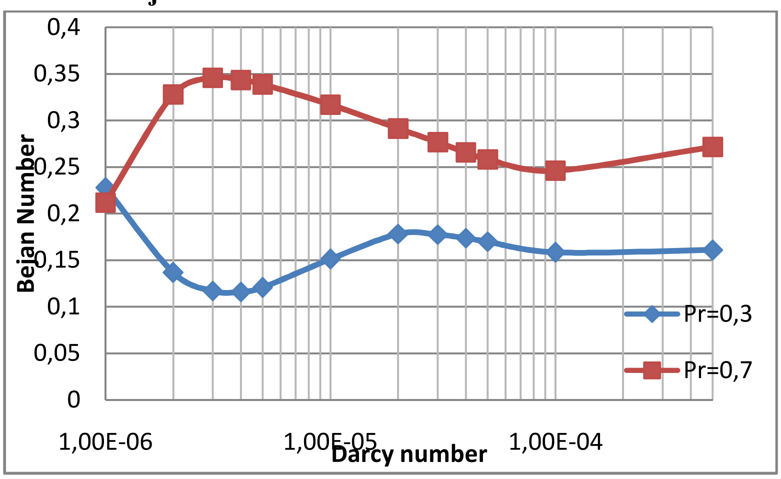


Figure 3 . Variation of Bejan number versus Darcy number for different Prandlt in model 2

The evolution of the Bejan number variation as a function of Darcy number is represented in the figure 3 for different Prandlt . It illustrates that at low Darcy number less than 2 10^{-6} the variation of Bejan for Pr=0,3 increase however for Pr=0,7 the variation of Bejan decrease .while Da more than 2 10-6 the variation of Bejan decrease progressively to attempt Da 10^{-4} .

4.4. Darcy number effect for Bejan number in model 3

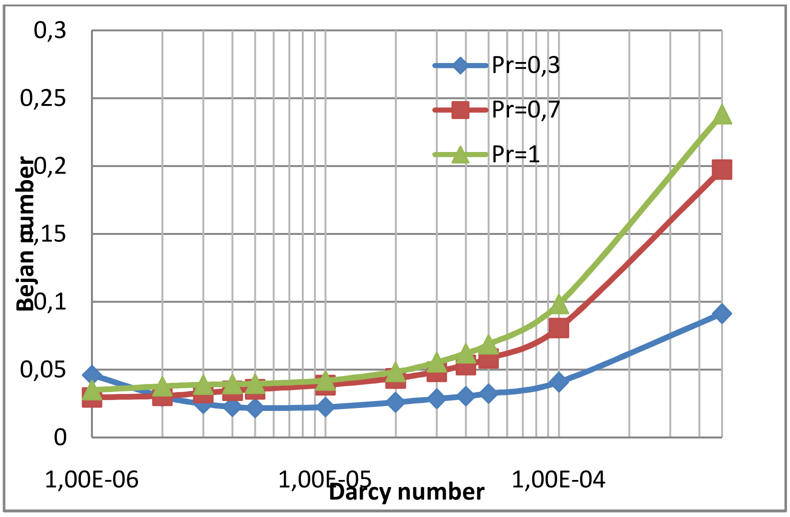


Figure 4: Variation of Bejan number versus Darcy number for different Prandlt in model 3

As shown in fig.4 the same variation of model 1. We can conclude that the presence of term of Niled in the viscous dissipation term have not a remarkable effect.

For low Darcy, the thermal energy in the system due to the viscous dissipation, than it starts to increase progressively.

4.5. Darcy number effect for Bejan number in model 4

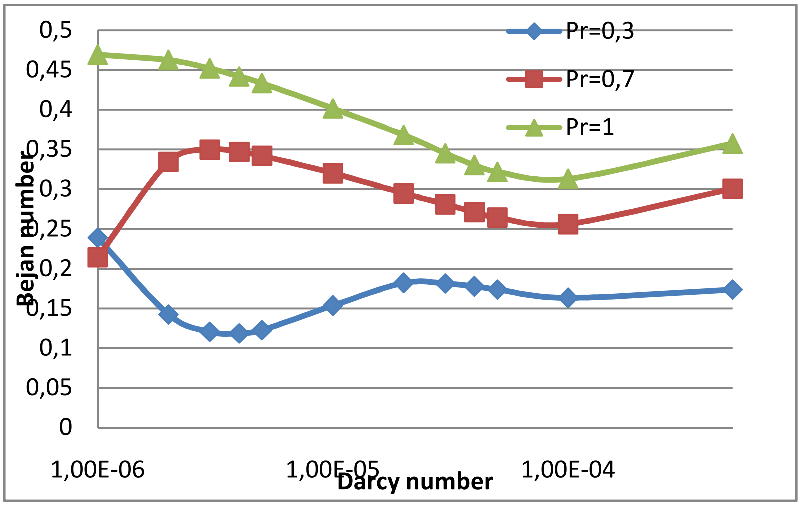


Figure .5: Variation of Bejan number versus Darcy number for different Prandlt in model 4

As illustrated in figure 3 the irreversibility in the system due to the viscous effect for less Darcy number . fig 5 have the same variation of model 2 with light variation we can conclude that the effect of Nieled term in the viscous dissipation dominate .

The thermal energy in the system is principally due to the viscous dissipation phenomena as long as a heat source which is pronounced as low Darcy numbers.

Conclusion:

Viscous dissipation terms effect has been studied in square cavity filled with porous media with a numerical program written in COMSOL Multiphysics software, to investigate the difference between four models :

a) Model 1: C1=0; C2= 0 (Nield and El-Hadhrami terms are neglected)
b) Model 2 :C1=1; C2=0 (Nield term is include)
c) Model3: C1=0; C2=1 (El-Hadhrami term is include)
d) Model 4: C1=1; C2=1 (Nield and El-Hadhrami are include)

The most important results are:

The presence of Niled term in the energy equation can improve our result so we can't neglect Nield term it have a persist effect for the irreversibility.

For less Darcy number the irreversibility in the system due to the viscous effect.

Prandlt influence for the production of entropy generation and it varied with models.

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