Modeling and Simulation of the Forced Convection in Horizontal Channel Partially Filled with Anisotropic Porous Matrix

Salah BEN AOUA [(1)] (bensalah101@gmail.com)

Rachid KIBBOUA [(2)], Farid MECHIGHEL [(1)]

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Introduction

A substantial part of theoretical and experimental investigations of heat transfer by convection in porous media have treated the case of isotropic materials[3, 4]. However, in many practical situations, porous materials are anisotropic in mechanical as well as in their thermal properties. However, improved heat transfer is always accompanied by an increase in pressure drop, and it is then necessary to find a compromise between these two parameters by making a judicious choice of thermo-physical properties of these media and the thickness of the porous layer.

The main objective of this study was to investigate the effect of hydrodynamic anisotropy on the flow and heat transfer rate in the channel under constant heat flux on the walls.

Analysis and modeling

The physical system studied here is illustrated in the figures 1(a) and (b) for the both configurations; in the first configuration, the porous medium is placed on the lower and upper surfaces of the channel, and in the second it is placed at the center of the channel.

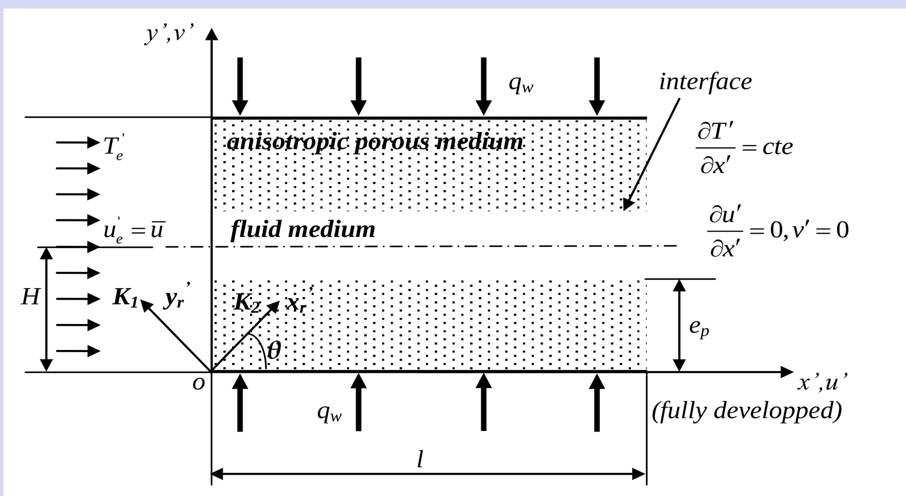

Figure 1. (a) Physical model and coordinate system (configuration 1).

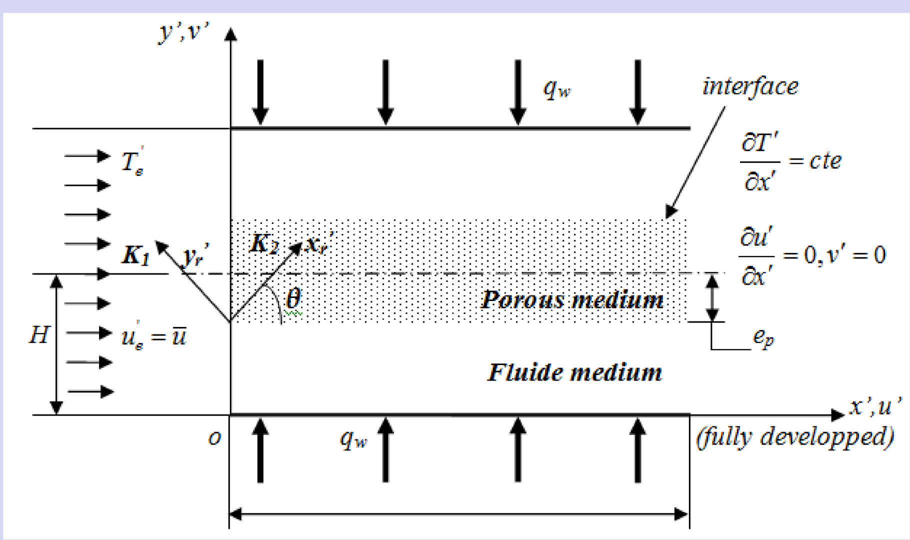

Figure 1. (b) Physical model and coordinate system (configuration 2).

In the present model, we adopt the following assumptions:

For the flow field: the flow is steady laminar and two-dimensional, the fluid is Newtonian and incompressible, and no volume forces were considered.

For the thermal field: the radiation heat transfer is negligible; neither viscous dissipation nor internal heat source were considered. Furthermore, the fluid is in *local thermal equilibrium* with the solid matrix and the thermal properties for both the fluid and the solid matrix are constants.

Governing equations

To formulate the governing equations we use the Navier-Stokes equations in the fluid region and the Darcy-Brinkman-Forchheimer equation in the porous region, these equations can be reformulated in the main reference system (x', y') using rotating matrices connecting the components of the velocity vector of a coordinate system (x_r', y_r') to a coordinate system (x ' , y ').

Dimensionless form of the governing equations

We consider the following dimensionless coordinates and variables:
$$\begin{cases} x = x'/H,\ y = y'/H \\ u = u'/\bar{u},\ v = v'/\bar{u} \end{cases} \text{and} \begin{cases} p = p'/\rho_f \bar{u}^2 \\ T = (T' - T_e')/\Delta T',\ \Delta T' = q_w H/k_f \end{cases}$$
$$\mathrm{Re} = \bar{u}H/\upsilon_f,\ \upsilon_{eff} = \mu_{eff}/\rho_f,\ \upsilon_f = \mu_f/\rho_f$$

Thus in dimensionless form the governing equations are:

Fluid flow:

For both the fluid and the porous region the dimensionless continuity equation is as follows: $\partial u/\partial x + \partial v/y = 0$ (1)

In the fluid region
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad (2.a)$$
$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \quad (2.b)$$

In the porous region
$$\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= -\varepsilon^2\frac{\partial p}{\partial x} + \varepsilon^2\frac{J}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \\ &- \left[\frac{\varepsilon^2}{\mathrm{Re}\,Da}\left(K^*\cos^2\theta + \sin^2\theta\right) + \frac{F\varepsilon^2}{\sqrt{Da}}\left(\sqrt{K^*}\cos^2\theta + \sin^2\theta\right)\sqrt{u^2+v^2}\right]u \\ &- \left[\frac{\varepsilon^2}{\mathrm{Re}\,Da}\left(K^*-1\right)\sin\theta\cos\theta + \frac{F\varepsilon^2}{\sqrt{Da}}\left(\sqrt{K^*}-1\right)\sin\theta\cos\theta\sqrt{u^2+v^2}\right]v \end{aligned} \quad (3.a)$$
$$\begin{aligned} u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} &= -\varepsilon^2\frac{\partial p}{\partial y} + \varepsilon^2\frac{J}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \\ &- \left[\frac{\varepsilon^2}{\mathrm{Re}\,Da}\left(K^*-1\right)\sin\theta\cos\theta + \frac{F\varepsilon^2}{\sqrt{Da}}\left(\sqrt{K^*}-1\right)\sin\theta\cos\theta\sqrt{u^2+v^2}\right]u \\ &- \left[\frac{\varepsilon^2}{\mathrm{Re}\,Da}\left(K^*\sin^2\theta + \cos^2\theta\right) + \frac{F\varepsilon^2}{\sqrt{Da}}\left(\sqrt{K^*}\sin^2\theta + \cos^2\theta\right)\sqrt{u^2+v^2}\right]v \end{aligned} \quad (3.b)$$

Energy equation:

In the fluid region
$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\mathrm{Pr\,Re}}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \quad (4)$$

In the porous region
$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{R_c}{\mathrm{Pr\,Re}}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \quad (5)$$

Dimensionless form of the boundary conditions:

The boundary conditions in dimensionless form are written as:

For: $x = 0 \quad \begin{cases} u = 1, \quad v = 0 \\ T = 0 \end{cases}$ (6.a) $\quad x = \frac{l}{H} = L \begin{cases} \partial u/\partial x = 0, \quad \partial v/\partial x = 0 \\ \partial T/\partial x = 0 \end{cases}$ (6.b)

$y = 0 \quad \begin{cases} u = 0,\ v = 0 \\ \partial T/\partial y = -1/Rc \end{cases}$ (7.a) $\quad y = 1 \begin{cases} \partial u/\partial y = 0,\ v = 0 \\ \partial T/\partial y = 0 \end{cases}$ (7.b)

$$y = e_p/H = E \quad \begin{cases} u_f = u_p,\ v_f = v_p \\ \frac{\partial u_f}{\partial y} = R_\mu\frac{\partial u_p}{\partial y},\ \frac{\partial v_f}{\partial y} = R_\mu\frac{\partial v_p}{\partial y} \\ T_f = T_p,\ \frac{\partial T_f}{\partial y} = R_c\frac{\partial T_p}{\partial y} \end{cases} \quad (8)$$

also, the *coefficient of friction* for fluid flow in dimensionless form is defined as:
$$f(\mathrm{Re}) = 8\,\mathrm{Re}(-\partial p/\partial x) \quad (9)$$

For the case of a flow in the *partially* filled channel of a porous medium, we define the *Nusselt number* as follows:
$$Nu = 4Hq_w/k_f\left(T_w' - T_m'\right) \quad (10)$$

To solve the system of coupled equations and the boundary conditions, we used a numerical method based on the finite volume method [5]. The SIMPLE algorithm is used for coupling the continuity and momentum equations. The system of equations obtained in matrix form is solved by the Thomas algorithm (TDMA). The resolution of the equations was performed on a uniform mesh 201 × 51, (201 nodes in the axial direction and 51 in the vertical direction). In order to verify the reliability of the code, a good agreement is made between our numerical results and the numerical or analytical results in the literature [4].

Results

All calculations were carried out using the values of the following parameters:

Aspect ratio (L/H=50); Porosity (ε=0.9); Prandtl number (Pr=0.7); Reynolds number (Re=100); Ratio of the thermal conductivities (R_c=1 and R_c=10); Kinematic viscosity ratio (J=1); Forchheimer coefficient (F=0); Darcy number (Da) between (10^{-4} and 10^{-1}); Thickness of the layer (E=0 "fluid", 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1 "fully porous medium"); Angle of anisotropy (θ = 0°, 30°, 60°, 90°, 120°, 150° and 180°)

Study of the configuration 1 (Case where the porous medium is placed on the lower and upper surfaces):

Variation of the friction coefficient with the thickness of the porous layer E: The figure 1 shows that the friction coefficient is an increasing function of the thickness (*E*). For a given thickness (*E*), the highest coefficient of friction corresponds to a ratio of permeability(K^*>1).

Variation of the Nusselt number as a function of the thickness of the porous layer (E): The figure 2, with the same conditions; the Nusselt number decreases with the increase in the thickness *(E)* to reach a minimum value for a given thickness *(E)*. This optimum thickness depends on the ratio of permeability (K^*). In contrast for (*R_c=10)*, the Nusselt number becomes an increasing function in terms of the thickness of the porous layer *(E)*, for a given thickness *(E)*, the most greater Nusselt number is obtained for a permeability ratio *(K^*<1)*, such as the values are most greater than the case of (*R_c=1)* (Fig.3).

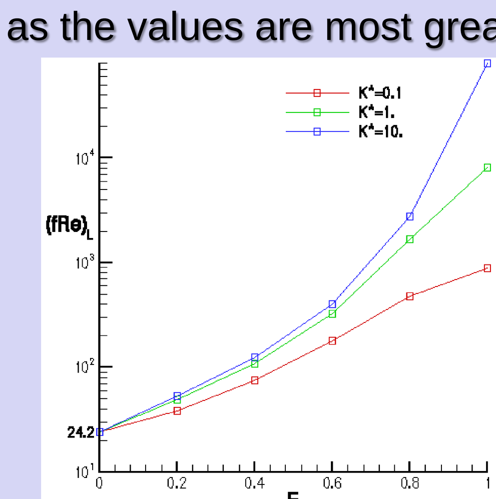

Figure 1: Variation of the friction coefficient with the thickness of the porous layer *E* for different K^*, *θ* = 0°, and *Da* = 10^{-3}.

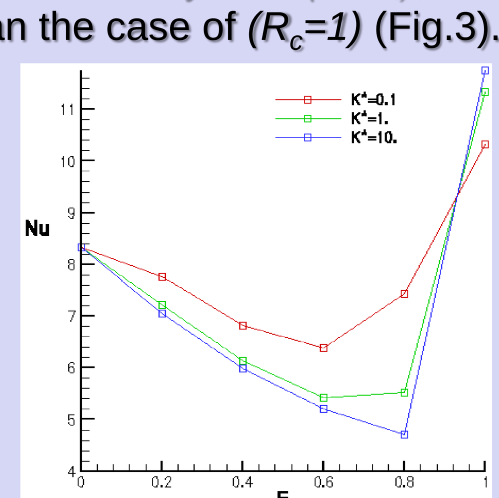

Figure 2: Nusselt number as a function of the thickness of the porous layer *E* for different K^*, *θ* = 0°, *Da* = 10^{-3}, R_c=1.

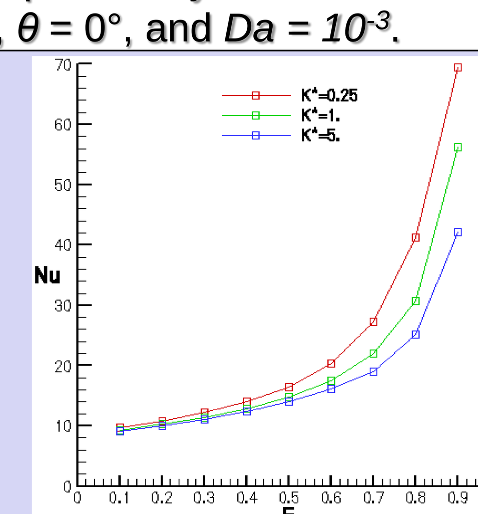

Figure 3: Nusselt number as a function of the thickness of the porous layer *E* for different K^*, θ = 0 °, Da = 10^{-3}, R_c= 10.

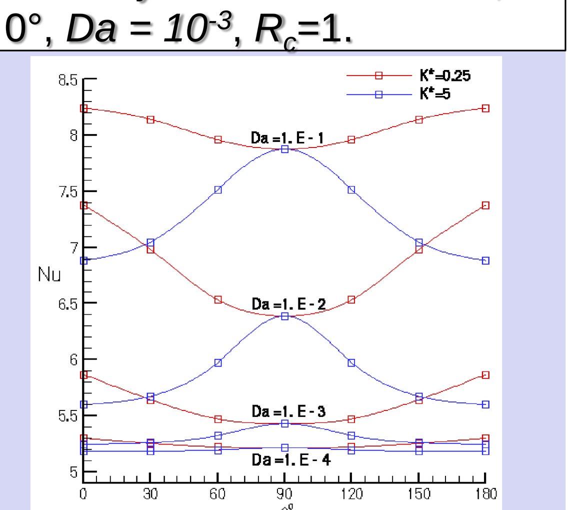

Figure 4: Effect of Darcy number Da and the angle of anisotropy on the Nusselt number for (K*=0.25 and K*=5), and (E=0.6).

Effect of Darcy number and the angle of anisotropy on the Nusselt number: In Figure 4, the thickness of the porous layer is fixed (E=0.6), the results indicate that for K*=0.25, the convective transfer is minimum with 90°, but it is maximum at 0° and 180°. The inverse is true for K*=5.

Combined effect of the angle of anisotropy, the Darcy number Da and the thickness E on the friction coefficient: Figure 5 shows that the coefficient of friction varies in proportion to the thickness *E* and also with the angle θ, for (K*=0.25).

The combined effect of the thickness and the anisotropy angle with K*=0.25 is depicted in Figure 6. The Nusselt number decreases with the increase of the thickness *E* until a minimum value for a given thickness *E*, this tendency reverses exceeding this thickness and exceeds the purely fluid case.

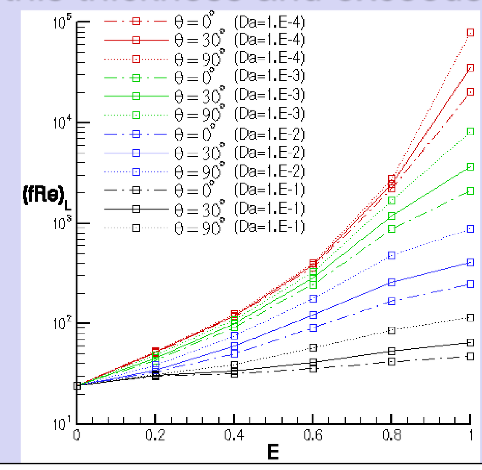

Figure 5: Combined effect of the angle of anisotropy, the Darcy number Da and the thickness *E* on the friction coefficient in the established region, for ( ).

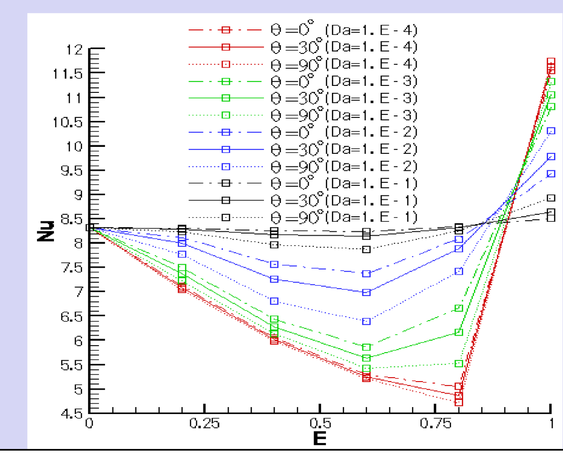

Figure 6: Combined effect of the angle of anisotropy, the Darcy number Da and the thickness *E* on the Nusselt number in the established region for ( ).

Study of the configuration 2: – Case where the porous medium is placed at the center of the channel:

Variation of the friction coefficient with the thickness of the porous layer (E): the friction coefficient is an increasing function of the thickness of the porous layer (*E*). For a given thickness, the highest friction coefficient corresponds to a ratio of permeability (*K^*>1)* that is to say when the permeability in the direction of flow is lower than that of the transverse direction (Figure. 7).

Variation of the Nusselt number as a function of the thickness of the porous layer E: Figure 8 shows that the Nusselt number increases with the increase of the thickness *(E)* until a maximum value for a given thickness *(E)*. This optimum thickness depends on the permeability ratio *(K^*)*. The best number of Nusselt number is with *(K^*>1)*.

For a thermal conductivity ratio *(R_c=10)*, the Figure 9 shows that the Nusselt number is higher than *(R_c=1)*.

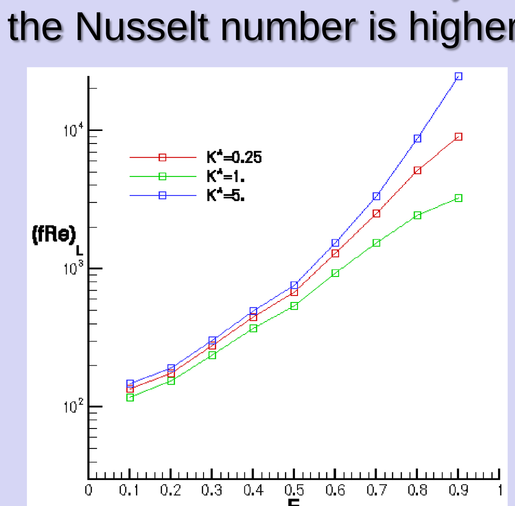

Figure 7: Variation of friction coefficient on the thickness of the porous layer *E* for different values of K^*, *θ* = 0°, *Da* = 10^{-3}.

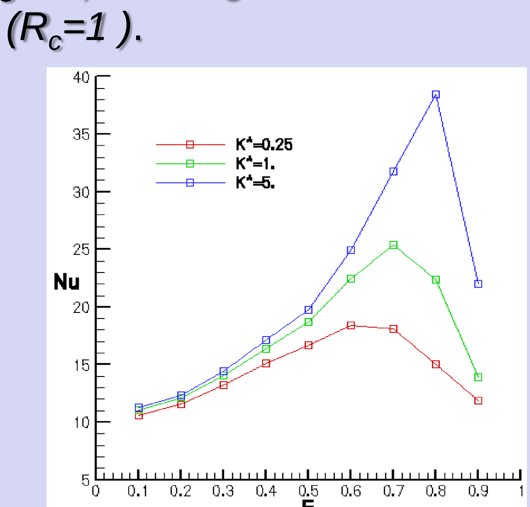

Figure 8: Variation of Nusselt number in terms of the thickness of the porous layer *E* for different values of K^*, *θ*=0°, *Da*=10^{-3}, R_c=1.

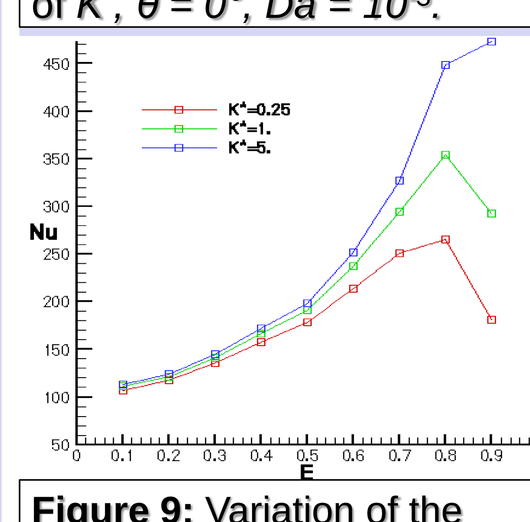

Figure 9: Variation of the Nusselt number in terms of the thickness of the porous layer *E* for different values of K^*, *θ* = 0°, *Da* = 10^{-3}, R_c = 10.

References

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Conclusions

-The use of a partially porous channel reduce the coefficient of friction and increase the heat transfer coefficient in the case *R_c>1* (mainly *Rc=10)*, and for second configuration; the Nusselt number increases even for *Rc=1* .

-For a given thickness *(E)* and a zero angle of orientation, the highest friction coefficient corresponds to a ratio of permeability *K^*>1*; and for an orientation angle of *90°*, the highest friction coefficient corresponds to a ratio of permeability *K^*<1*; that is to say when the permeability in the direction of flow is lower than the transverse direction.

-For the first configuration, the Nusselt number decreases with the increase in the thickness *(E)* to reach a minimum value for a given thickness *(E)* when R_c=1 .

-When R_c=10, the Nusselt number becomes an increasing function of the thickness of the porous layer *(E)*. For a given thickness *(E)*, the highest Nusselt number is obtained for a ratio of permeability *K^*<1*.

-For the second configuration, the Nusselt number increases with the increase of the thickness *(E)* until a maximum value for a given thickness *(E)* is reached. With a ratio of *K^*>1* gives rise to better heat transfer.

-Finally, the second configuration is better than first in terms of a thermal viewpoint but leads a coefficient of friction slightly higher.

[(1)] Laboratory LR3MI, Mechanical engineering department, faculty of sciences of engineering (UBMA), BP 12, Annaba 23000, Algeria.
[(2)] Laboratory LTPMP, Faculty of Mechanical Engineering and Process Engineering (USTHB), BP 32 El Alia, Algiers 16111, Algeria.