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MRT-LBM simulations of natural convection coupled to surface radiation in a square cavity cooled from above and locally heated from the vertical walls

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Abstract: The present study is dedicated to a 2D-modeling of natural convection coupled with radiation in a square cavity partially heated from the sides and cooled from above. The computation of the fluid flow and heat transfer was carried out using the Lattice-Boltzmann method with the MRT collision scheme. The study was performed for a wide range of the internal surfaces emissivity ($0 \leq \varepsilon \leq 1$). The remaining parameters are the Rayleigh number, Ra = 10^6, the Prandtl number, Pr = 0.71 and the temperature difference ΔT = 30 K. The results obtained are presented in terms of streamlines, isotherms, heatlines and Nusselt numbers. Preliminary tests of validation were successfully conducted and show that the numerical code is well adapted to treat efficiently such problems. This study revealed that the emissivity of the walls has a determining effect both on the flow structure and the heat transfer generated by the heating sources.

Keywords: Natural convection, surface radiation, partial heating, MRT scheme, Lattice-Boltzmann method.

1. Introduction

The study of natural convection in closed rectangular cavities still arouses much interest given its omnipresence in daily life and in many industrial applications such as cooling of electronic components, thermal building, metallurgical industry, etc. An important part of the literature in this field is summarized in books by Bejan [1] and Yang [2]. Besides, numerous investigations that deal with convective heat transfer inside enclosures with different shapes and thermal excitations were conducted during the past few decades. Nevertheless, the study of natural convection combined with radiation constitutes a challenge for researchers doing experimental and theoretical works. In fact, in many applications involving natural convection as a removal of heat transfer mechanism, thermal radiation is present and could be neglected only if the involved temperatures are low and/or when the walls of the confining cavities have low emissivities, which is not the case in practice. Hence, several works have been performed on the coupled between natural convection and surface radiation, for a more reasonable approach of thermal convection problem. These studies have shown that radiation affects both dynamical and thermal characteristics of the working fluids and leads in general to substantial improvement of the total heat exchange between the confining boundaries. Among the earliest studies on the coupling between natural convection and thermal radiation in rectangular cavities, Larson and Viskanta [3] have examined the effect of surface radiation when coupled to laminar natural convection. Their results indicated that radiation dominates the heat transfer in the enclosure and alters significantly the convective flow patterns. Akiyama and Chong [4] analyzed numerically the interaction of natural convection with surface radiation in an air-filled square cavity. Their results show that the presence of emissive surfaces has a negligible effect on the average convective heat transfer while the radiative heat transfer rapidly increases with the emissivity of the internal surfaces. Moreover, Wang et al. [5] studied the effect of surface radiation on natural convection in a square cavity bounded by isothermal vertical walls and adiabatic horizontal walls. Detailed analysis showed that the net radiative heat flux varies linearly either with the imposed maximum temperature difference or with the dimensional height of the cavity. In the literature, most of the previous works devoted to problems involving natural convection coupled to surface radiation focused on rectangular cavities with horizontal or vertical temperature gradients. However, situations where the walls may be subject to various types of thermal non-uniformities are numerous and encountered in many practical applications. In this context, El Ayachi et al. [6] studied numerically the combined effects of radiation and natural convection in a square cavity submitted to two modes of cross gradients of temperature. There results showed that the first mode of heating (bottom wall heated and upper one cooled) leads to an important intensification of the flow circulation. The second mode (bottom wall cooled and upper one heated), with a basically stable vertical gradient temperature, leads to a slowdown of the fluid circulation. According to the same study, the contribution of radiation to the total heat transfer is generally important as much as the emissivity of the active parts of the internal surface is higher and the imposed temperature difference is important. Most of the existing numerical studies have solved such problems using conventional methods such as finite difference and finite volume methods. Thus, the objective of this work is to experiment the Lattice-Boltzmann method with the MRT scheme to address a convection-radiation coupling problem in a square cavity partially heated from the lower parts of the vertical walls and cooled from above.

2. Mathematical Formulation

2.1 Problem description

The configuration under study is sketched in Figure1. It consists of a square cavity filled with air, cooled from above and partially heated from the vertical sides. The heated portions are placed at the lower half of the vertical walls. The remaining parts of the internal surface are considered adiabatic. Initially, the fluid was considered at a rest state with a uniform temperature $T_0 = (T_h + T_c)/2$. The study is performed for a wide range of the emissivity ($0 \leq \varepsilon \leq 1$) by setting constant the Rayleigh number (Ra = 10^6), the Prandtl number (Pr = 0.71) and the temperature difference (ΔT = 30 K). The reference temperature is taken to be $T_0 = 298\ K$.

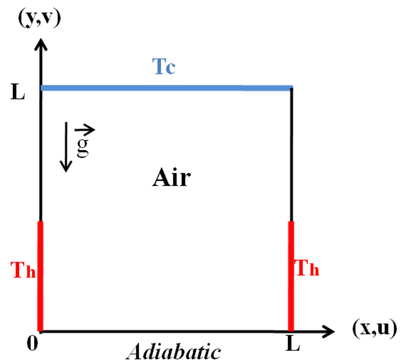


Figure 1: Studied configuration

2.2 Lattice Boltzmann method

The Lattice-Boltzmann method [7] used to simulate the fluid flow is based on the resolution of the following Boltzmann equation:

$$f(r + c\Delta t, t + \Delta t) - f(r,t) = \Omega(f) \qquad (1)$$

The MRT scheme considers that the propagation phase, corresponding to the left-hand part of the above equation, occurs at the microscopic level in the space formed by the discrete velocities $c_k\,(k = 0,..,8)$ while the collision phase, represented by the operator Ω in Eq. (1), takes place in a macroscopic space formed by the moments of the distribution functions. The transition between these two spaces is insured by the following matrix M [8]:

$$m = Mf \qquad (2)$$

In the collision phase, the non-conservative moments undergo relaxations to an equilibrium state with different rates (MRT: Multi-Relaxation-Time) according to the relation:

$$m^* = m - s(m - m^{eq}) \quad (3)$$

The conserved moments, mass and momentum, are modified by the presence of the buoyancy force as follows:

$$m_\rho^* = m_\rho = \rho \quad (4)$$

$$m_u^* = m_u = \rho u \quad (5)$$

$$m_v^* = m_v = \rho v + F_y \quad (6)$$

In natural convection, the buoyancy force is deduced based on the Boussinesq approximation that considers density linearly depending from the temperature with the thermal expansion coefficient as a slope:

$$F_y = \rho g \beta (T - T_{ref}) \quad (7)$$

After the propagation phase that persists during Δt, macroscopic values are calculated from the new distribution functions:

$$\rho = \sum_{k=0}^{8} f_k = m_0 \quad (8)$$

$$u = \frac{1}{\rho} \sum_{k=0}^{8} f_k . c_{x,k} = \frac{m_u}{\rho} \quad (9)$$

$$v = \frac{1}{\rho} \sum_{k=0}^{8} f_k . c_{y,k} = \frac{m_v}{\rho} \quad (10)$$

The boundary conditions are treated with the standard bounce-back scheme which insures the impermeability and no-slip of the fluid on the solid walls.

The temperature is a scalar variable that can be assimilated to the fluid density. Thus, the energy equation can be solved using a thermal version of the MRT scheme considering a reduced lattice model; the D2Q5 with only five discrete velocities describing the possible streaming directions of the thermal distribution function g. This thermal model also consists of a series of collision and propagation phases after which the temperature field is obtained from the zeroth integration of the thermal distribution function over the directions of the D2Q5 model:

$$T = \sum_{k=0}^{4} g_k \quad (11)$$

The inner surface of the cavity was divided into elementary surfaces having at their centers the border nodes of the selected mesh. The radiosity J_{rd}, the convection-radiation interaction parameter N and the view factors of these elements are determined adopting the same approach as that in Raji et al. [9]. In the following, the results will be presented at a non-dimensional scale.

The average convective Nusselt numbers along the top wall and the vertical heated portion are given by the following expressions, respectively:

$$Nu_{cv}^{T} = \frac{1}{L}\int_0^L \frac{\partial T}{\partial y}\Big|_{y=L}, \qquad Nu_{cv}^{L,R} = \frac{1}{L}\int_0^L \frac{\partial T}{\partial x}\Big|_{x=0,L} \quad (12)$$

The average radiative Nusselt numbers along the top wall and the vertical heated elements are respectively given by the following expressions:

$$Nu_{rd}^{T} = \frac{1}{L}\int_0^L N J_{rd}|_{y=L}, \qquad Nu_{rd}^{L,R} = \frac{1}{L}\int_0^L N J_{rd}|_{x=0,L} \quad (13)$$

In the oscillatory regime, the mean Nusselt numbers Nu_t are obtained by time averaging Nu_{cv} and Nu_{rd} over one flow cycle. The total Nusselt number stands for their sum.

3. Results and discussion

3.1 Validation of the numerical code

A part of this work is dedicated to the validation of the numerical code based on the MRT scheme. Our results were compared against those of El Ayachi et al. [6] obtained with a numerical code based on a finite difference method to study the coupling between natural convection and thermal radiation in a square cavity submitted to partial heating and cooling. The Prandtl number, Rayleigh number and emissivity of the walls were fixed at Pr = 0.72, Ra = 10^6 and ε = 0.5, respectively. Figure 2 shows quantitative and qualitative comparisons between our results and those of El Ayachi et al. [6]. These comparisons show an excellent agreement, with a maximum relative difference in terms of Nusselt Number of about 1.11 %. Thus, it can be concluded that the numerical code elaborated to study the coupling between natural convection and surface radiation, is well adapted to treat efficiently such problems.

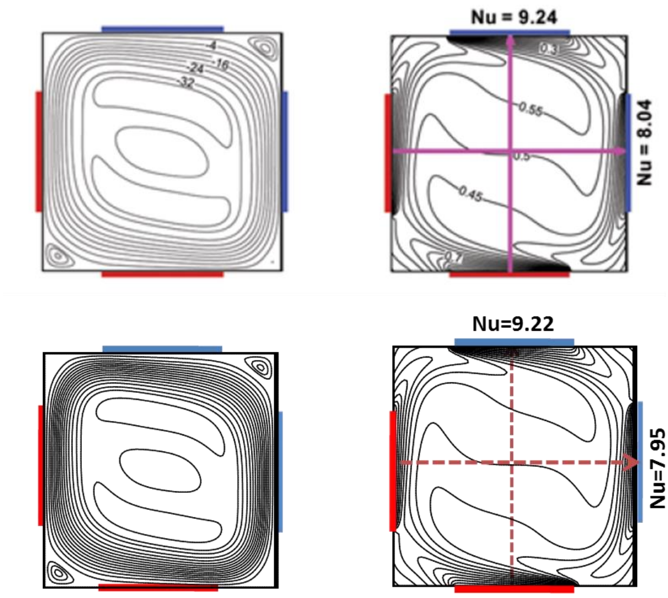


Figure 2: Comparison of the streamlines and isotherms obtained with our numerical code against those from ref. [6] for ε = 0.5 and Ra = 10^6.

The preliminary simulations carried out on our configuration showed the existence of an unsteady flow regime. Therefore, the results from the grid dependence test reported in table 1 correspond to time averaged values. From this table, the noticed maximum relative difference when the grid passes from 100×100 to 150×150 is about 0.74%. It is about 0.13% when an extra refinement is considered from 150×150 to 200×200. Consequently, the refinement of the grid above 100×100 has a negligible effect on the obtained results and the latter grid was selected to deal with the present problem.

Table1: Effect of the grid size on the mean Nusselt number at the cold wall and maximum of stream-function for Ra = 10^6 and ε = 0.5

Grid size	Nu_{cv}	Nu_{rd}	$\lvert\psi\rvert_{max}$
100×100	6.38	4.50	38.70
150×150	6.41	4.51	38.41
200×200	6.43	4.51	38.46

3.2 Walls' emissivity effect on the fluid flow

In this subsection, the effect of the emissivity of the walls on the flow structure is considered. As mentioned before, the preliminary simulations have shown the existence of both steady and unsteady flows depending on the value of the emissivity. Figure 3 illustrates the streamlines, isotherms and heatlines for the steady state flow in the case of pure convection (a), and for relatively low values of the emissivity (b, c) that are below the critical value leading to the unsteady regime. In figure 3a, the flow is dominated by two horizontal cells with a small vortex located in the lower-right corner of the cavity, showing the strong resistance of the upper positive cell to the breakdown under the effect of the negative one. By taking into account the effect of radiation, it can be observed from figures 3b and 3c that the bicellular flow structure is conserved. However, the small increase of the emissivity of the walls, has engendered a significant reduction of the positive cell intensity in favor of that of the negative one. Lines corresponding to heat flux (heatlines) show a similarity with the streamlines and demonstrate the dominance of convective heat transfer over conduction mode (high values of the stream function).

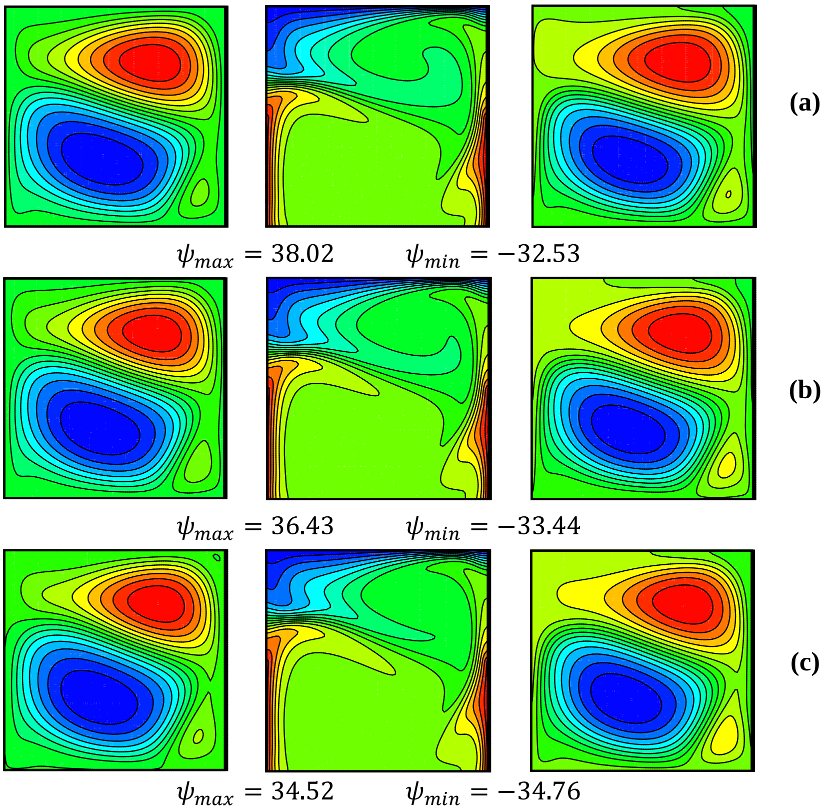


Figure 3: Streamlines, isotherms and heatlines for the steady regime: (a) ε = 0, (b) ε = 0.025 and (c) ε = 0.05.

By increasing progressively the emissivity of the walls while maintaining the Rayleigh number at $Ra = 10^6$, an unsteady flow regime appears from a critical value of ε (around $\varepsilon = 0.1$). Above this critical value of ε, the convection regime turns unsteady and the oscillations are periodic in time with relatively high amplitudes. This behavior is illustrated in figure 4, for ε = 1, by presenting temporal variations of ψ_{max} and ψ_{min}. As it can be seen from this figure, the evolution of ψ_{max} vs. time is the same as that of ψ_{min} in absolute value with a phase shift of roughly a half period (T = 0.1617). The corresponding streamlines and isotherms at selected instants in figure 4 during a flow cycle are presented in figure 5 to illustrate the changes undergone by the flow structure. According to this figure, the flow structure evolves between a vertical bicellular flow (P1-P5) and a horizontal bicellular flow (P2) passing from their mirror images with respect to the vertical (P3) and horizontal (P4) mid-lines, respectively. In addition to the main cells, small vortices appear intermittently at the corners of the cavity. Over this flow cycle, a remarkable competition between the two dominating cells is observed. After being heated from the adjacent heat source (right cell in P1/left cell in P3), one of these cells becomes lighter and moves above the other one (positive cell in P1/ negative cell in P3) trying to establish a good contact as large as possible with the upper cold wall. At this time, the above cell (positive cell in P1/negative cell in P3) discharge its heat load through the upper cold wall while the lower cell (negative cell in P1/positive cell in P3) is getting heated from the heat sources. As a result, the above cell/(lower cell) becomes heavier/(lighter) and tends to move down/(up) (P3) announcing the start of the second half of the flow cycle whose structure is symmetric to that in the first half. For such a case, the period of the Nusselt number on the upper cold wall/(heating portions) is expected to be the half of/(same as) the flow period.

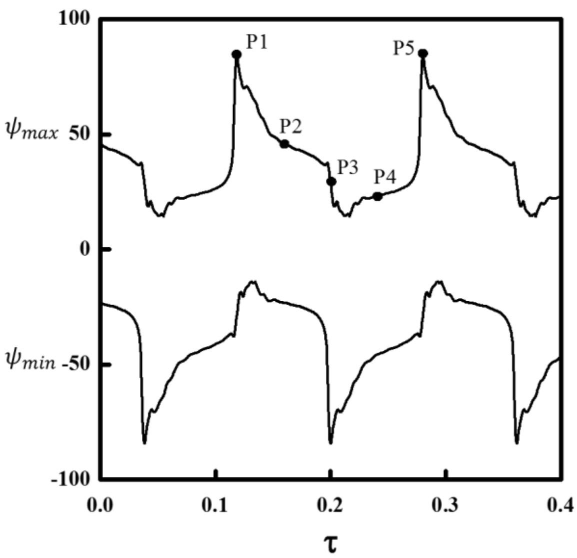
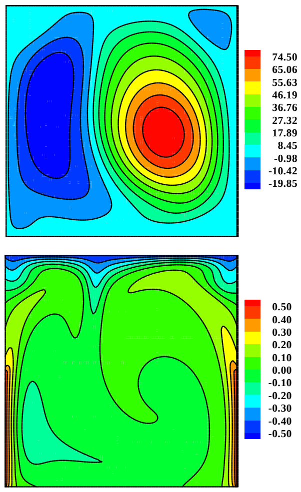
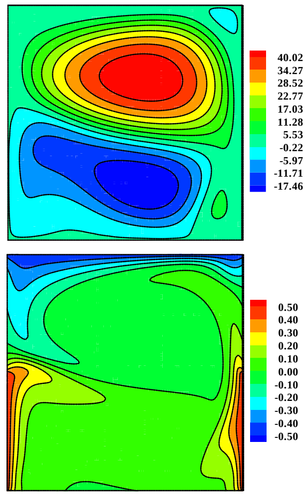


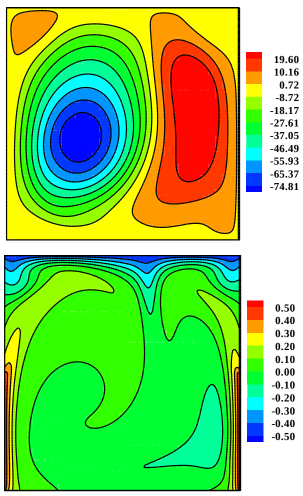
Figure 4. Temporal variations of ψ_{ext} for $\varepsilon = 1$.



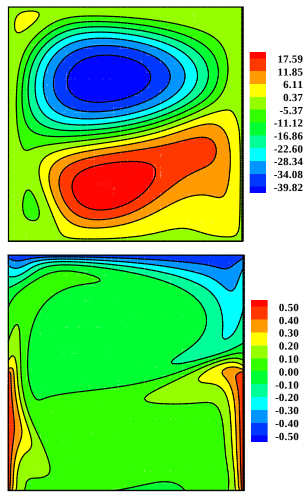
P1 (or P5): $\psi_{max} = 83.93$, $\psi_{min} = -29.29$



P2: $\psi_{max} = 46.76$, $\psi_{min} = -23.21$



P3: $\psi_{max} = 29.04$, $\psi_{min} = -84.25$



P4: $\psi_{max} = 23.33$, $\psi_{min} = -45.56$

Figure 5. Streamlines and isotherms at selected instants of the flow cycle.

3.3 Walls' emissivity effect on heat transfer

In the present subsection, attention is focused on the quantification of natural convection and radiation contributions to the overall heat transfer. Figure 6 shows the variations of the mean convective, radiative and total Nusselt numbers (time- averaged values of one flow cycle) versus the emissivity of the walls at the cooled and the heated walls of the cavity. From figure 6a, we note that, due to the dimension of the active portions, the top wall leads to higher convective heat transfer in comparison with that corresponding to each of the heating sources. The convective Nusselt number at the top wall increases with the wall's emissivity up $\varepsilon = 0.5$ above which it starts to decrease slightly. Also, above this threshold value of ε, the vertical walls contribute with the same amount to the convective heat transfer. From figure 6b, we can observe that the radiative heat transfer of the left wall is less important than that of the right one for $\varepsilon \leq 0.5$ (the role of the vertical walls is inverted if the image solutions are considered). Above this value of ε, the vertical portions contribute equitably to the radiative heat transfer but with amounts lower than that of the top wall. The effect of ε on the total heat transfer is presented in figure 6c. It can be seen from this figure that the total Nusselt number on each active wall is characterized by a linear increase with ε. More precisely, an improvement of the total Nusselt number on the left wall of about 193 % is noticed when the value of the emissivity passes from 0 to 1. Similarly, the total heat transfer is enhanced by almost 165 % for the right wall and 169 % in the case of the cold wall. These results show that radiation plays a non-negligible role in the heat transfer process since it contributes by important amounts of heat. Its effect could be neglected only for very particular situations (case of polished surfaces for instance).

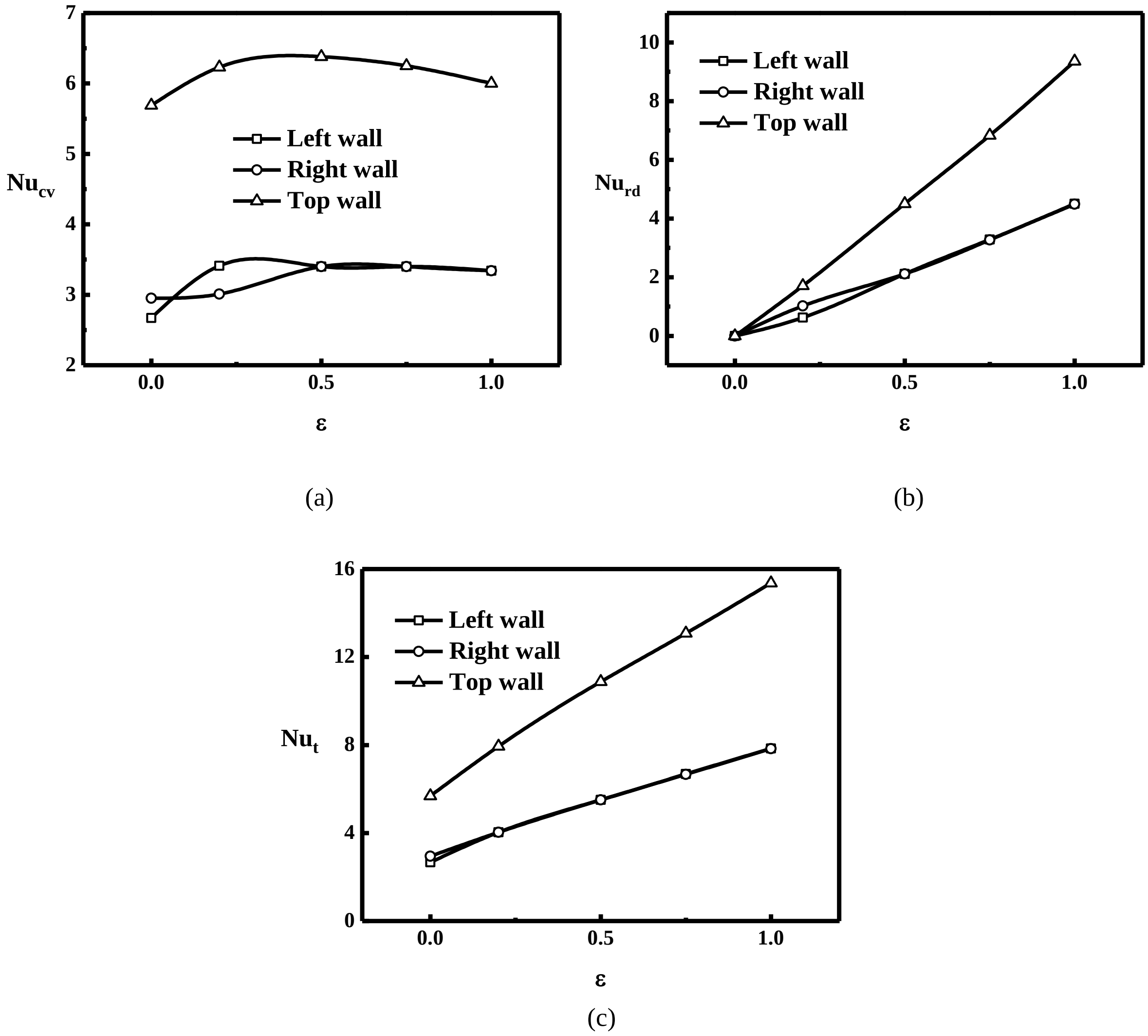


Figure 6: Variations of the convective (a), radiative (b) and total (c) Nusselt numbers on the active portions versus ε for Ra = 10^6.

The contribution of radiation to the total heat transfer through the active walls of the cavity is quantified by presenting, in figure 7, the evolution of the ratio Rad = Nu_{rd}/Nu_t with ε. The ratio evolution is characterized by a monotonous increase with the emissivity of the walls. The contribution of radiation to the overall heat transfer is seen to become important and could not be neglected even for low to moderate values of the walls' emissivity.

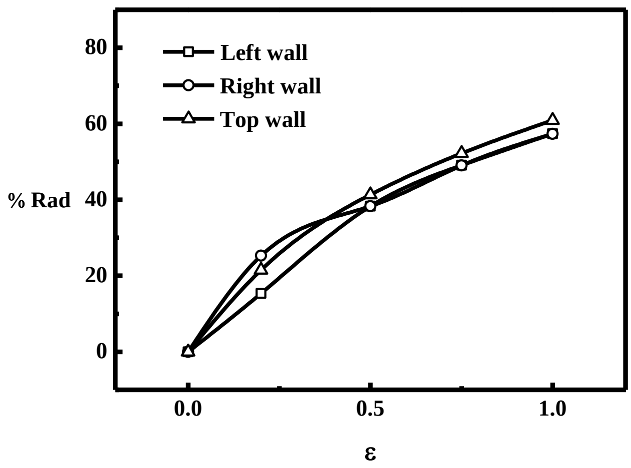


Figure 7: Effect of ε on the contribution of radiation to the total heat transfer.

Conclusion

A numerical investigation of natural convection coupled with surface radiation in a square cavity was carried out by adopting the MRT scheme of the Lattice-Boltzmann method. The half-lower parts of the vertical walls are heated and the upper horizontal wall is cooled. The obtained results show that the emissivity of the walls favors the transition from steady to unsteady periodic solutions from a critical value of ε around 0.1. This means that the study of such problems would be dramatically truncated if the effect of radiation is neglected. The flow structures observed are characterized by a strong competition between the main positive and negative cells and the nature of this competition is realistically emphasized by examining the effect of ε. The numerical simulations revealed also that radiation contributes largely to the enhancement of the overall heat transfer and should be considered.

Nomenclature

f	vector of distribution functions
g	vector of distribution functions relative to temperature
F_y	buoyancy force
g	gravitational acceleration, *m/s*^{2}
m	vector of the moments of distribution functions
m^*	vector of moment after the crash phase
m^{eq}	vector of moments in equilibrium
J	non-dimensional Radiosity
N	convection-radiation interaction parameter
Pr	Prandtl number
Ra	Rayleigh number
Δr	distance between two neighboring D2Q9 network nodes
s	vector of the relaxation rate
Δt	time step
T	temperature, K
T_c	temperature of the cold wall, K
T_h	temperature of the hot wall, K
T_{ref}	reference temperature, K
ΔT	difference temperature, K
u	x component of velocity
v	y component of velocity

Greek symbols

β	expansion coefficient, K^{-1}
ε	emissivity (radiation)
Ω	collision operator
ψ	stream function
ρ	density, *Kg/m*^{3}

Subscripts

cv	convective
rd	radiative
t	total

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