

Natural convection in vertical and inclined rectangular enclosures containing adiabatic fins staggered attached on the heated and cold walls

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Abstract

The natural convection heat transfer in inclined rectangular enclosures with adiabatic fins staggered mounted on the hot and cold walls is numerically studied using the energy and Navier-Stockes equations with the Bossiness approximation. The parameters governing this problem are the Rayleigh number ($10^3 \le R_a \le 10^5$), the geometric parameter($0.25 \le C \le 1$), the relative length of the fins(0.25 < B < 0.75), The aspect ratio of the enclosure ($3 \le A \le \infty$) and Pr = 0.72 the inclination angle from the horizontal was from 0 to 60 degree, the variation of the local Nusselt number Nu_{loc} along the enclosure height and the average Nusselt number Nu as a function of Ra are computed. Streamlines and isotherms in the enclosure are produced. The results show that B is an important parameter affecting the heat transfer trough the cold of the cavity. The heat transfer is reduced for increasing inclination angle. The results show that the heat transfer can generally be reduced by using appropriate geometrical parameters in comparison with a similar enclosure without fins.

Key words:

Natural convection, numerical study, heat transfer, fins

1. Introduction

Natural convection in enclosures is a topic of a several studies, simple enclosures, defined as chambers with no partitions in them, have been studied in the past, but current interest has now shifted to complex enclosures containing partitions such as cavities with fins which represent a prototype of many industrial applications like reactor insulation, ventilation of rooms, solar energy collection and cooling of devices.

Since a review of literature shows that there is a great number of studies on simple enclosures with various wall conditions have been extensively considered by researchers [1-4], studies of complex enclosures will be the primary topic of discussion herein .Heat transfer analysis in cavities with an array of rectangular blocs has also received significant consideration in recent years[5-7].Heat transfer enhancement in cavities due to incorporation of fins attached to the walls has a great interest last years[8-15]. Natural convection in the vertical and inclined enclosure with adiabatic fins attached on the heated wall was numerically studied by Hasnaoui[16], the aspect ratio was from 2.5 to ∞ , the dimensionless fin length was from 0 to 1, the

Rayleigh number was from $10^4 \le R_a \le 2 \times 10^5$ it was observed that fin length was an important parameter: at low Rayleigh numbers, the heat transfer was reduced with increasing fin length but at high Rayleigh numbers there was an optimum value. Heat transfer was reduced for decreasing micro cavity height and passed from a maximum for an inclination angle. Lakhal et al [17] studied numerically a similar enclosure with perfectly conducting fins. it was concluded that the heat transfer through the cover is considerably affected by the presence of the conducting fins. Natural convection in an air filled, differentially heated, inclined square cavity, with a dia-thermal partition on its cold wall was numerically studied by Frederick[18].at Rayleigh numbers of 10³-10⁵. The imposed partition caused convection suppression, and heat transfer reductions of up 47% relative to the undivided cavity at the same Rayleigh number .what is more a finite-volume-based numerical study of steady laminar natural convection(using Bossinesq approximation) within a differentially heated square cavity due to the presence of a single thin fin was investigated by Shi and Khodadadi [19] the results show that heat transfer is lower with shorter fins and that its growth for perfectly conducting fins is marginal for lengths exceeding half the cavity side. They also reported augmentation in heat transfer performance when the thin fin attached to the hot wall was positioned closer to the insulated walls.Meanwhile,Nag et al.[20] had studied natural convection heat transfer in a differentially heated square cavity with a horizontal partition plate placed on the hot left wall. The plate was alternated to examine both infinite thermal conductivity and insulated conditions the Rayleigh number was between 10^3 - 10^6 while three partition lengths placed at three positions were analyzed. it was concluded that for a partition of infinitely high thermal conductivity, the Nusselt number on the cold wall was greater than the case with no partition regardless the position of the partition on the hot wall. .FengXu et al.[21] reported the effect of the fin length on natural convection transition in a cavity. They showed that the transition from a steady flow to unsteady flow is sensitive to the fin length.Bilgen [22] investigated natural convection in differentially heated square cavites with a thin fin on the hot wall, the range of Rayleigh number employed was between 10^{4} - 10^{9} , dimensionless fin length between 0.1 and 0.9, dimensionless fin position between 0 and 0.9 and dimension fin conductivity ratio between 0(perfectly insulating) and 60 the results of study indicated that the average Nusselt number was an increasing function of Rayleeigh number and a decreasing function of fin length and thermal conductivity ratio, they also showed that heat transfer was enhanced when the fin length was short and positioned near the insulated horizontal walls for the case with conductivity ratio of one. In addition Ben-Nakhi and Chamkha[23]studied numerically conjugate natural convection in a square enclosure with a perfectly conductive inclined fin attached to the heated wall. Three different fin lengths equal to 20%, 35% and 50% of the heated surface length were considered. It was found that the thin fin inclination angle and length, and solid-to-fluid thermal conductivity ratio have significant effects on the local and average Nusselt number at the heated surfaces of the enclosure/fin system. The investigation implied that the presence of an inclined thin fin reduces the average Nusselt number at the heated surfaces in nonordered configuration.

The case of cavities containing several adiabatic fins staggered mounted on the hot and cold walls, remains still relatively less documented. The main goal of this investigation is to study heat transfer by natural convection in these cavities and to drive useful correlations for the design and heat transfer in such systems.

2. Mathematical model

The schematic drawing of the system with geometrical and boundary conditions is shown in fig 1.the height and the width of the inclined enclosure is denoted by H' and L', respectively. The depth of the enclosure perpendicular to the diagram is assumed to be long. Hence, the problem can be considered to be two-dimensional.the top and the bottom walls are adiabatic ,whereas the right wall is maintained

at height temperature (T'_{H}) and the right wall is maintained at lower temperature (T'_{C}) , with $(T'_{C} \leq T'_{H})$ several adiabatic horizontal fins staggered with length l'_{p} are mounted on the two vertical walls with an equal distance h'_{p} from each other.



Figure 1: Schematic of the problem, the coordinate system and boundary condition

The bossinesq approximation is adopted for the variation of the density in the buoyancy term. The equations describing the dynamics and the thermal fields are those of Navier-stokes coupled with the equation of energy transport. in a non dimensional form they are written as :

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial X} + \Pr(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}) + Ra.\Pr.T.\sin\varphi$$
(2)

$$\frac{\partial V}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \Pr(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}) + Ra.\Pr.T.\cos\varphi$$
(3)
$$\frac{\partial T}{\partial T} + U \frac{\partial T}{\partial Y} + V \frac{\partial T}{\partial Y} = \frac{\partial^2 T}{\partial Y} + \frac{\partial^2 T}{\partial Y}$$

$$\overline{\partial t} + \mathcal{O} \, \overline{\partial X} + \mathcal{V} \, \overline{\partial Y} = \overline{\partial X^2} + \overline{\partial Y^2} \tag{4}$$

Dimensionless form of the governing equations was obtained via introducing dimensionless variables. These are defined as follows:

$$X = \frac{x'}{L'} \qquad X = \frac{y'}{L'} \qquad U = \frac{u'.L'}{\alpha} \qquad V = \frac{v'.L'}{\alpha}$$

$$P = \frac{p'.L^2}{\rho\alpha^2} \qquad T = \frac{T'-T_c'}{T_H'-T_c'}$$
(5)

Parameters Ra and Pr are the Rayleigh and prandtl numbers, respectively. These are defined as:

$$Ra = \frac{g\beta(T'_H - T'_C)}{v\alpha^2} \qquad \Pr = \frac{v}{\alpha}$$
(6)

The dimensionless form of the boundary conditions:

On the left wall (X=0) U=V=0 , T=0 On the right wall (X=1) U=V=0 , T=1 On the bottom and top walls (Y=0 and Y=A) U=V=0 , $\frac{\partial T}{\partial Y} = 0$ (7) On the fins U=V=0 , $\frac{\partial T}{\partial Y} = 0$

The quantities of practical interest are the local and the overall Nusselt numbers .At the cold wall, the local heat flux leaving the system is given by:

$$Nu_{loc} = \frac{hL'}{k} = \frac{\partial T}{\partial x}\Big|_{x=0}$$
(8)

The normalized average Nusselt number is obtained as

$$Nu = \frac{\overline{h} L'}{k} = \frac{1}{A Q_c} \int_0^A \frac{\partial T}{\partial x} \Big|_{x=0} dy$$
(9)

Where Q_c is the overall dimensionless heat transfer by pure conduction. This gives the net heat transfer rate leaving the system through the cover.

An other variable utilized to evaluate the heat transfer rate for the left and right walls of the cavity is the Nusselt number ratio NNR, with its defined as:

$$NNR = \frac{Nu_{with.a.fin}}{Nu_{without.fin}}$$
(10)

3. Numerical method

3.1 Algorithm

The finite volume-method using the SIMPLER algorithm (Semi Implicit Method for Pressure Links Equation Revised [10] was used to solve the governing equations specified in the previous section. Hybrid scheme was utilized to evaluate the convective terms whereas the central differences was used for the diffusive terms. The evolutions with time of the Nusselt number, ψ_{max} and ψ_{min} were examined for each program run to make sure that final state is steady or unsteady.

The accuracy of the numerical code was verified by comparing results from the present investigation with those of the benchmark solutions compiled by vahl Davsis[1] and xundai shi [19].table [1] show that the agreement is very good.

Table 1:	Comparative	results in terms	s of the average	Nusselt number for	a square cavity	differentially heated
	1		U		1 2	2

Ra	Nu[1]	Nu[19]	Nu[this study]
10 ⁴	2.243	2.247	2.247
10 ⁵	4.519	4.532	4.5324
10 ⁶	8.80	8.893	8.8735

It also reproduces ,with an excellent agreement ,the results of natural heat transfer in a differentially heated square cavity due to a thin fin on the hot wall(xundai shi and khodadadi) [19] as table 2 illustrate:

Table 2: Comparative results in terms of the maximum stream function and the Nusselt Number ratio on the right wall (T=1), for a square cavity with a thin fin on the hot wall

	length	$\psi_{\rm max}$ [19]	$\psi_{\rm max}$ (this study)	NNR [19]	NNR (this study)
$RA = 10^4$	L=0.2 L=0.35 L=0.5	4.51 3.8 2.66	4.51058 3.75169 2.6272	0.62 0.46 0.33	0.61628 0.452 0.3229
$RA = 10^5$	L=0.2 L=0.35 L=0.5	9.65 10.22 10.42	9.64815 10.17772 10.35707	-	0.718 0.63 0.5712
$RA = 10^{6}$	L=0.2 L=0.35 L=0.5	17.91 18.01 17.9	17.9123 18.00573 17.9715	- -	0.794 0.746 0.713

As a further check, the average Nusselt numbers at the hot and cold walls were compared, which showed a maximum difference less than 0.5 % in all runs, hence all the energy furnished by the hot wall to the fluid leaves the cavity by the cold wall.

3.2 Grid independence study

In order to determine the proper grid size of this study, a grid independence tests was conducted for the Rayleigh number of 10^5 in rectangular enclosure containing several adiabatic fins staggered attached to the hot and cold walls. the length of the fins was set to be 50 percent of L', the geometrical ratios are C=1 and A=5. The obtained results, presented in table [3], show that the results obtained with the grid of 120 x600 (grid adopted in this study) are in agreement with those obtained with the most finer grid (200 x1000).the maximum deviations observed are within 0.21% and 0.23% Respectively in terms of the Nusselt number Nu and the maximum stream function ψ_{max} across the active walls, Time step was between 0.0004 et 0.0008.

Table 3: Variation of the average Nusselt number Nu and maximum stream function ψ_{max} according to the grids for Ra=10⁵

Grid	$\psi_{ m max}$	Nu	
60x300	12.41614	3.34636	
120x600	12.46653	3.319	
200x1000	12.49447	3.312	

4. Results and Discussion

4.1 Local Nusselt number

Flow and temperature fields, and heat transfer rates for various Rayleigh numbers, Ra, and geometric ratios A,B and C will be examined in the following section. All analyses were carried out assuming air as fluid with Pr =0.72 and for the following conditions :the aspect ratio A = H'/L' is varied from 3 to ∞ .the spacing between adjacent fins is (C=0.5,C=1,C=1,5,C=2) and the dimensionless height of the fins is (B=0.25,B=0.5,B=0.75).the range of Rayleigh number is 10^3 - 10^5 .the effects of dimensionless fin length B, Spacing between fins C ,and title angle φ are discussed later.



Figure 2:Local Nusselt number variation on the cover along the height (y) for A=3,C=1, $\varphi = 0$ and various



Figure 3: Local Nusselt number variation on the cover along the height (y) for A=5, $C=1, x\phi = 0$ and various Ra and B.



Figure 4: Local Nusselt number variation on the cover along the height (y) for A=10,C=1, $\varphi = 0$ and various Ra and B.

The local Nusselt number, Nu_{loc} along the cover is calculated using Eq.8 for various B and Ra and presented in Fig.2-4 for A=3 ,5 and 10 respectively. The limiting cases of rectangular enclosure without fins(B=0) are also shown in each case ,figures 2-4 show that B is an important parameter affecting the heat transfer ,its influence increase with increasing Ra .generally the local Nusselt number Nu_{loc} is oscillatory and increasing along the height of the enclosure (increasing y), whose amplitude increase with increasing Ra .the local Nusselt number Nu_{loc} increase with decreasing B at the top of the cover but decrease with decreasing B at the bottom of the enclosure.

Figures 2-4 generally show similar for various A. however the sinuous variation of the local Nusselt number Nu_{loc} is more pronounced for larger A. these results can be explained by examining the streamlines(on the left) and the isotherms (on the right) in Fig.5.all values of ψ are positive indicating that the fluid circulation is clockwise. the sinuous variation of Nu_{loc} observed in Fig.3 are caused by the contraction of the isotherms almost in front of fins at low Ra (Fig.5) and at the space under fins at higher Ra

The general circulation of the fluid in the cavity is decreased with increasing B(longer fins)and the conduction regime is more established .at higher values of Ra $(5.10^4 \text{ and } 10^5)$ the general fluid circulation is increased with decreasing B

It can also be seen from the streamlines presented that the general circulation of the fluid is in the micro-cavities for B=3/4 and it is more and more in the space between the end of the right fins and the end of left fins as B decreases.

For B=3/4 the cavity has many micro-cavities each operating the same conditions except the first and the last ones. For B<3/4 the flow interaction between the cavity and the micro-cavities makes the local heat transfer smoother.

d	6	t	d	e	Ť
g	h	i	j	k	I

Figure 5: a-1 Steady-state streamlines (on the left) and isotherms (on the right) for A=5,C=1, $\varphi = 0$ a B=1/4 Ra=10³,b B=1/2 Ra=10³,c B=3/4 Ra=10³,d B=1/4 Ra=10⁴,e B=1/2 Ra=10⁴,f B=3/4 Ra=10⁴ g B=1/4 Ra=5x10⁴,h B=1/2 Ra=5x10⁴,i B=3/4 Ra=10⁵,j B=1/4 Ra=10⁵,k B=1/2 Ra=10⁵,l B=3/4 Ra=10⁵

4.2 Effect of Rayleigh number

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In order to study the effect of the effect of the fins on the average heat transfer rate, we introduced a variable called the Nusselt number ratio (NNR). The NNR can be obtained according to Eq.10, value of the NNR greater than 1 indicates that the heat transfer rate is enhanced on the surface , wheras reduction of the heat transfer is indicated when NNR mean Nusselt number is less than 1.



Figure 6: Nusselt number ratio as a function of Ra for $\varphi=0$, C=1, and various Ra and B

Normalized Nusselt Number Ratio (NNR) as function of Rayleigh number Ra, for C=1 and different values of B and A, is presented in Fig.6.Based on this figure, it is observed that the NNR become smaller with the increase of the fin's length regardless of the Rayleigh number. This is because the fins can block the flow near the fins and reduce the corresponding convection in that area. upon comparing the diagrams in Fig.6.we can see that the effect of the fins is less remarkable with the rise of the Rayleigh number .further more Fig.6 show that the NNR is increasing with increasing A.

4.3 Effect of the number of micro-cavities

the cross plot of Nu as a function of 1/n(n represent the number of microcavities) for various B and Ra numbers is shown in Fig.7.it is seen that the average Nu is quasi constant at all Ra numbers. the heat transfer is dominated by conduction. The variation of Nu with B is weak When A increase from 3 to ∞ .the values of Nu are higher for B=0.25 at all Ra numbers.



Figure 7: average Nusselt number variation versus 1/n for $\phi=0$,C=1,and various Ra and B

4.4 Effect of angle ϕ on heat transfer

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The tilt angle ϕ was varied from zero (vertical cavity) to 60° from the horizontal (Fig.1) and similar computations were carried out for the case of A=5 and A=10, The results are presented in Fig.8 as average Nu as a function of the tilt angle ϕ for B=1/4 and 3/4 at Ra=10



Figure 8: average Nusselt number variation versus ϕ for C=1,A=5 and various Ra and B

The results show that the heat transfer decreased with increasing inclination angle of the tall enclosure. For $Ra=10^4$ the variation of Nu is weak which means that the heat transfer was dominated by conduction .The effect of becomes more important for high Rayleigh number ($Ra=10^5$).

4.5 The effect of C on the heat transfer

The geometrical parameter $C=h_P'/H'$ was varied from 0.25 to 1 with A=5.the results are presented as average Nu as a function C for B=3/4,1/4 and Ra=10⁵ and 10⁴ in Fig.9.it can be seen that the heat transfer is increasing function of C and that this increase is more enhanced at high Rayleigh numbers. the dominant heat transfer is by conduction for small C. the convection becomes visibly dominant for larger C.



Figure 9: Average Nusselt number variation versus C for A= 5, ϕ =0 ° and various Ra and B

The streamlines and isotherms of the case of A=5 and $Ra=10^5$ are shown in Fig.10.It is shown that for C=0.25, the heat transfer is mainly by conduction with the isotherms parallel to the end walls and very little convection in the micro-cavities. For C=1.0 Fig.10 shows that the convection is enhanced by the increased general circulation as well as the circulations in the micro-cavities resulting in high heat transfer.



Figure 10 :Steady-state streamlines (on the left) and isotherms (on the right) for A = 5, B = 3/4, ϕ = 0 °, Ra = 102. a C = 0.25, b C = 1.00

Conclusion

The heat transfer in inclined rectangular enclosures with adiabatic fins staggered mounted on the hot and cold walls was numerically studied. It is found that flow regime at low Ra is one of pure conduction. At heigh Rayleigh numbers, the heat losses through the cold wall are reduced in enclosure containing attached fins. The parameters governing this problem are the dimensionless fin length *B*, the cavity aspect ratio *A*, the micro-cavity aspect ratio *C*, and the inclination angle φ

- i) It is seen generally that heat transfer is reduced with increasing B
- ii) The heat transfer reduction is enhanced with increasing inclination angle ϕ
- iii) The effect of inclination angle is more important at high Rayleigh numbers
- iv) The NNR is increasing with increased A
- v) Then convection is enhanced with larger C

Nomenclature:

A	aspect ratio, L' / H'		Time		
В	dimensionless fin length, l_p' / L'		Temperature of fluid		
С	dimensionless distance between fins,	$\Delta T'$	Temperature difference		
h_{p}^{\prime}/L	L'	<i>u'</i> , <i>v'</i>	Velocities in x' and y' directions		
g	Acceleration due to gravity $(m.s^{-2})$	<i>x</i> ′, <i>y</i> ′	Cartesian coordinates		
h	local heat transfer coefficient		symbols		
\overline{h}	- h Average heat transfer coefficient on heated		Thermal diffusivity		
wall		β	Volumetric coefficient of thermal		
h_{-}'	Distance between fins(m)	expansi	on		
p		φ	The angle of the inclination		
H'	<i>I'</i> Total height of cavity(m)		Kinematic viscosity		
l_p'	Length of fins(m)	ρ	fluid density		
L'	Total length of cavity(m)	τ	Dimensionless time		
n	number of micro-cavites	Ψ	Dimensionless stream function		
Nu	mean Nusselt number		Subscripts		
NNR	Nusselt number ratio	С	cooled wall		
Pr	prandtl number,[$ u/lpha$]	Н	heated wall		
Ra	Rayleigh number $[g\beta\Delta TL^{6}/(\nu\alpha)]$	Loc	local		
t	dimensionless time, $t'/(L^2/\alpha)$	Super s	script		
-			Dimensional variables		

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