ACCURATE SOLUTIONS OF 3D NATURAL CONVECTION FLOW

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ABSTRACT :

Accurate solutions of the equations governing the natural convection of air in a cubic cavity, thermally driven on two opposite vertical faces are given for Rayleigh number values up to 10^7. These solutions are obtained with a pseudo-spectral Chebyshev algorithm based on the projection-diffusion method [1,2] with a spatial resolution supplied by a 81x81 polynomial expansion. The solutions are believed to be accurate to better than [0,002, 0,02]% in relative spatial error for the corresponding Rayleigh number (Ra) range [10^3, 10^7]. They clearly indicate a non monotonous evolution of the flow structure as Ra increases.

Key words : natural convection, cubic cavity, air, spectral approximation.

1. INTRODUCTION

A good knowledge of the natural convection flows in differentially heated enclosures is a valuable starting point for testing and validating computer codes used for a wide varieties of practical problems, such as cooling of radioactive waste containers, ventilation of rooms, solar energy collectors and crystal growth in liquids. In the design of such devices, De Vahl Davis and Jones [3,4] have published in 1983 a bench-mark numerical solution of the buoyancy-driven flow in a square cavity with differentially heated vertical sides, the other ones being adiabatic, for a Rayleigh number value lying in the range [10^3-10^6], and a Prandtl number fixed at 0.71. By resorting to a systematic grid refinement practise and by concurent use of the Richardson extrapolation to obtain grid-independent data, these solutions were claimed to be within an accuracy of 1%. In 1991, P. Le Quéré [5] proposed accurate numerical solutions, obtained with a pseudo-spectral Chebyshev algorithm, for Ra number values up to 10^8, that is very close to the transition to unsteadiness which occurs at Ra= 1.82 10^8. [6].

In the last two decades, owing to improvements in algorithms and computing ressources, three_dimensional flow calculations have been performed for the differentially heated (on two opposite vertical faces) cavity [2,7-13]. All together, these papers refer to several different 3D configurations. For the one which is considered here (the fluid is confined in the three space directions and the non active walls are adiabatic), to th author's knowledge, only the references [2, 9, 11-13] supply characteristic values of the stationary velocity fields and/or the Nusselt numbers, for values of the Ra number lying in the range [10^3-10^7]. It has been shown, recently, by [2] that the natural convection flow of air, in this configuration, becomes unsteady for a value of the Ra number situated just beyond this range, with an hysteretic behavior extending from 3.2 10^7 to 3.5 10^7.

The purpose of this communication is to complete the two- and the three-dimensional flow calculations database by providing five accurate solutions corresponding to Ra=10^3, 10^4, 10^5, 10^6 and 10^7 respectively. In particular, these data clearly indicate a non monotonous evolution of the flow structure as Ra increases.

2. THE MATHEMATICAL MODEL AND THE NUMERICAL METHOD

The usual dimensionless Boussinesq equations are :

$$\frac{\partial \hat{v}}{\partial t} + (\hat{v}\cdot\hat{\nabla})\hat{v} = -\hat{\nabla} p + Pr\nabla^2\hat{v} + Ra\ Pr\ T\ \hat{e}_z$$

$$\hat{\nabla}\cdot\hat{v} = 0$$

$$\frac{\partial T}{\partial t} + (\hat{v}\cdot\hat{\nabla})T = \nabla^2 T$$

Where :

- unite vector $\hat{e}_z$ points the upward vertical direction, $\hat{e}_x$ and $\hat{e}_z$ being the unit vectors in the horizontal plane,
- the lengths, the velocity $\mathbf{v} = u\ \hat{e}_x + v\ \hat{e}_y + w\ \hat{e}_z$ and the temperature T are scaled by H, the size of cube, the thermal diffusion velocity κ/H and the imposed temperature difference ΔT, respectively, the other scales being derived from these,
- Ra and Pr (fixed at 0.71) are the Rayleigh and Prandtl numbers.

No-slip boundary conditions are imposed on all the faces of the cube. The thermal conditions applied on the active faces are $T(x = 1/2,y,z) = -1/2$, $T(x = -1/2,y,z) = ½$, the other faces being adiabatic ($\partial T/\partial n = 0$, at (x, $y=\pm 1/2$,z) and (x,y, $z = \pm 1/2$) where $\partial/\partial n$ stands for the appropriate normal derivative. The 2D square configuration corresponds to the particualr case of flows which are invariant by translation along the $\hat{e}_y$ direction. They are often assumed to approximate the cross-section in the mid-plane y = 0 of the 3D flows which are invariant by reflection about this plane.

A Chebyshev Gauss-Lobatto method has been used to evaluate the fields spatial derivatives. The velocity and pressure have been uncoupled by a « projection-diffusion » approach, recently proposed in [1,2]. It is an unconditionaly stable direct solver of the unsteady Stokes problem discretized in time with a second order scheme. The pressure is first evaluated, form a Darcy system, to cancel the numerical value of the divergence of an intermediate field, which becomes in its turn the source of an advection-diffusion equation to be solved for the velocity field. The obtained divergence improves itself exponentially as the nodes numner increases, as expected from the numerical divergence of any non singular velocity field. These equations have been time integrated by a classical second order finite differences scheme (Crank-Nicolson for the diffusion terms and Adams-Bashforth for any explicit evaluation).

3. THE NUMERICAL RESULTS

Convergence to steadiness is declared when the criterion

$$|\Phi_n - \Phi_{n-1}| / |\Phi_n|\ \delta t = 10^{-1}$$

is satisfied for all Φ_n standing for the maximum value, found on the nodes at time n δt, of one of the physical fields. 3D results at Ra=10^6, with $\delta t = 4\ 10^{-6}$, have been obtained with more severe criteria than the quoted one, by one to three decades. They are quite equivalent, all departing by 0.03% form the results obtained with the criterion 10^{-1}.

2D and 3D calculations have been performed, for each chosen Ra number value, applying the following procedure. For a given value of the Ra number, a steady solution has been first obtained on the coarser grid, starting either from from rest or from a flow corresponding to a smaller Ra number value. This solution was then projected onto a finer Gauss-Lobatto grid by a Lagrange interpolant. Time integration was then restarted using this extrapolated flow as initial condition and carried out until the new steady-state achievment. The process was repeated until the solution be obtained on the finest grid.

An excellent agreement has been observed between our 2D results and the benchmark data given by [4] and [5].

3D calculations have been performed at five Rayleigh number values (10^3, 10^4, 10^5, 10^6 and 10^7). For each one of these values, a 3D flow has been obtained with four different meshings : 51^3, 61^3, 71^3 and 81^3. The assessment of the accuracy of our results has been made at Ra=10^6 and 10^7 with additional flows obtained with seven other grid refinements. As the number of nodes increases, each quantity converges towards a given value (zero, in particular, for the divergence). The exponential decrease of the relative divergence, as a function of the nodes number, confirms the spectral improvement of the data. In order to evaluate the accuracy of our data we have calculated, for each maximum of the velocity component, a relative error with the quantity

$$ER[\Phi_{N1}] = |\Phi_{max,N1} - \Phi_{max,N2}| / |\Phi_{max,N1}| \quad ; N1<N2$$

Where $\Phi_{max,N1}$ stands for the absolute maximum of one the velocity components, calculated with the N^3 grid. The evolution of these quantities as a function of the nodes number suggests a spectral decreasing of the relative error on the velocity field. Thus, with the finest grid used here (81^3 nodes) the relative accuracy is within 0.02%. Repeating this procedure at $Ra=10^3$ and 10^4 with the four meshings indicated above, the corresponding relative error is evaluated at 0.002%. Thus, the space convergence can be considered as reached to supply accurate solutions.

4. BENCH-MARK SOLUTIONS

A comparison of our data with those already published [7-13] is realized and a first set of « bench-mark three-dimensional solutions » is proposed (Table 1). The quoted data are purposedly restricted to their significant digits. As already indicated in [2], all these flows are invariant by reflexion about the mid-plane (y=0). The measured relative symmetry rate is better than 10^{-9}.

The goal of this paper is not to characterize the 3D structure of the flows. Nonetheles, a noticeable feature emerges from the reading of Table1. Indeed, the mis-plane contains the maximum of the u velocity component, except for the $Ra=10^5$ and 10^6 cases, which suggests a non monotonous evolution of the flow structure when the Ra number value increases. This is confirmed by looking at thefollowing three relative heat transfer rates, $100x(Nu_{2D,W} - Nu_{3D,W})/Nu_{2D,W}$, $100x(Nu_{2D,W} - Nu_{mp})/Nu_{2D,W}$, $100x(Nu_{3D,W} - Nu_{mp})/Nu_{3D,W}$ whose Ra dependencies are shown on Figure 1. $Nu_{2D,W}$ is the global 2D Nusselt number at the wall , Nu_{3D} is the global 3D Nusselt number at the wall and Nu_{mp} is the mid-plane Nusselt number (y=0), whrer y is the coordinate along the horizontal « depth » direction $\hat{e}_y$.

As well known since the thermally driven cavity is studied, $Nu_{3D,W}$ is less than $Nu_{2D,W}$, at least until the onset of unsteadiness, their relative departure being maximum (9%) at about $Ra=10^4$. With respect to the $Nu_{2D,W}$, the mid-plane heat transfer increases, goes beyond it, until reaching a maximum (at about $Ra=10^5$) and then goes back to get close, but slightly larger, to $Nu_{2D,W}$. So, the 2D approximation of the natural convection flow of air overestimates the effective 3D heat transfer and, surprisingly, underestimates the mid-plane effective heat transfer rate. When $Nu_{2D,W}$ gives the worst estimation of $Nu_{3D,W}$, it is the best for Nu_{mp}.

Table 1 : Benchmark solutions. This Table contains the following quantities : (a) the maximum, in the cavity, of each velocity component (u,v,w), with its location, (b) the maximum, in the mid_plane (y=0), of the u and w components (v cancels there), with their locations, (c) the maximum, in the mid-plane (y=0), of the u components at x=0 and of the w component at z=0, and their locations, (d) the Nusselt numbers $Nu_{3D,W}$ and Nu_{mp}, and (e) d_{3D}, d_{mp} which are respectively the maximum in the cubic cavity of the nodal divergence, scaled by the larged modulus of the velocity, and the maximum in the cavity mid-plane (y=0) of (?v/?y), reached by the largest modulus of the velocity.

Rayleigh number					
	10^3	10^4	10^5	10^6	10^7
u_{max}	3.543	16.719	43.90	126.97	383.78
X	0.0166	0.0196	-0.1841	-0.3057	-0.3777
Y	$0.54\ 10^{-11}$	$0.12\ 10^{-10}$	0.2203	0.2997	$0.37\ 10^{-10}$
Z	0.3169	0.3250	0.3873	0.4365	0.4663
v_{max}	0.173	2.156	9.69	25.56	83.40
X	$0.14\ 10^{-10}$	0.3823	0.4175	0.4518	-0.3316
Y	0.2521	0.2826	0.3390	0.3983	0.4083
Z	$0.43\ 10^{-10}$	0.3447	0.3801	0.4168	0.3953
w_{max}	3.544	18.983	71.06	236.72	768.06
X	0.3223	0.3834	0.4304	0.4604	0.4775
Y	$0.38\ 10^{-10}$	0.2308	0.3736	0.4299	0.4601
Z	0.0032	0.0206	0.0060	0.0265	0.0323
$U_{mp,max}$	3.543	16.719	43.06	123.47	383.76
X	0.0166	0.0196	-0.1865	-0.3133	-0.3777
Z	0.3169	0.3250	0.3848	0.4366	0.4662
$W_{mp,max}$	3.544	18.682	65.43	218.25	698.44
X	0.3233	0.3870	0.4368	0.4638	0.4794
Z	0.0032	0.0219	0.0100	0.0353	0.0354
$U_{mp,max}(0,0,z)$	3.538	16.721	37.56	68.21	154.56
Z	0.3151	0.3244	0.3535	0.3536	0.3716
$W_{mp,\ max}\ (x,0,0)$	3.541	18.616	65.21	217.57	686.93
X	-0.3147	-0.3802	-0.4330	-0.4669	-0.4755
Nu_{mp}	1.087	2.250	4.612	8.877	16.547
$Nu_{3D,W}$	1.070	2.054	4.337	8.640	13.342
d_{3D}	$0.103\ 10^{-4}$	$0.280\ 10^{-3}$	$0.112\ 10^{-2}$	$0.363\ 10^{-2}$	0.032
d_{mp}	0.286	0.541	0.421	0.130	0.206

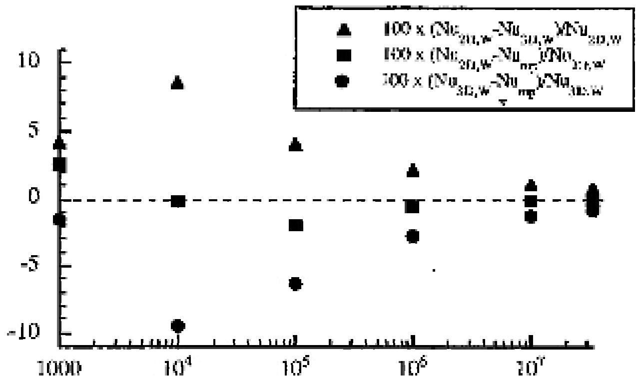

Fig. 1. The relative heat transfer rates as functions of the Rayleigh number

With respect, now, to Nu_{mp}, the 3D heat transfer is always weaker. The minimum it presents (by almost 10%, at about Ra=10^4) indicates a strong y-dependency of Nu(y). The last data, at Ra=3.3 10^7, come from the ultimate steady flow obtained before the onset of the unsteadiness reported in [2]. It has been added to the Bench-mark data in Figure 1 in order to clarify the heat transfer rate evolution at the boundary of the steady flows domain. Unsteadiness occurs before the 2D and 3D heat transfer rates become too close. The 3D structure of these flows deserves to be studied in detail.

5. CONCLUSION

Accurate solutions to the cubic differentially heated cavity problem have been presented, for values of Ra number in the range [10^3, 10^7], that is almost up to the end of the steady laminar regime. From mesh refinements and extrapolations, the spatial resolution of the data is believed to be better than 0.02% in relative spatial error at the highest Rayleigh number. A non monotonous dependency on the Rayleigh number of the flow structure emerges clearly from the quoted bench-mark data.

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